AQA Maths Decision 1

Past Paper Pack

2006-2015

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Wednesday 18 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 5 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 (a) Draw a bipartite graph representing the following adjacency matrix. *(2 marks)*

	U	***V***	***W***	***X***	***Y***	***Z***
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(b) Given that initially A is matched to W, B is matched to X, C is matched to V, and E is matched to Y, use the alternating path algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

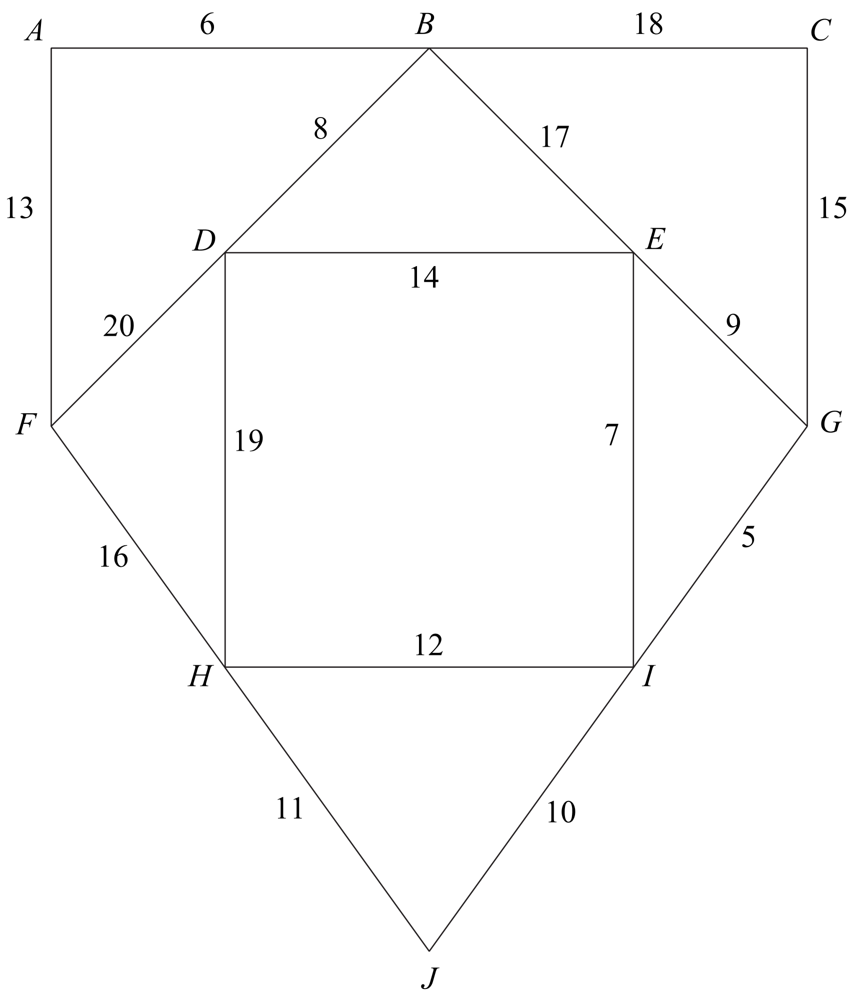
2 Use the quicksort algorithm to rearrange the following numbers into ascending order. Indicate clearly the pivots that you use.

18 23 12 7 26 19 16 24 *(5 marks)*

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, …, J. The numbers on each edge represent the distances, in miles, between pairs of vertices.

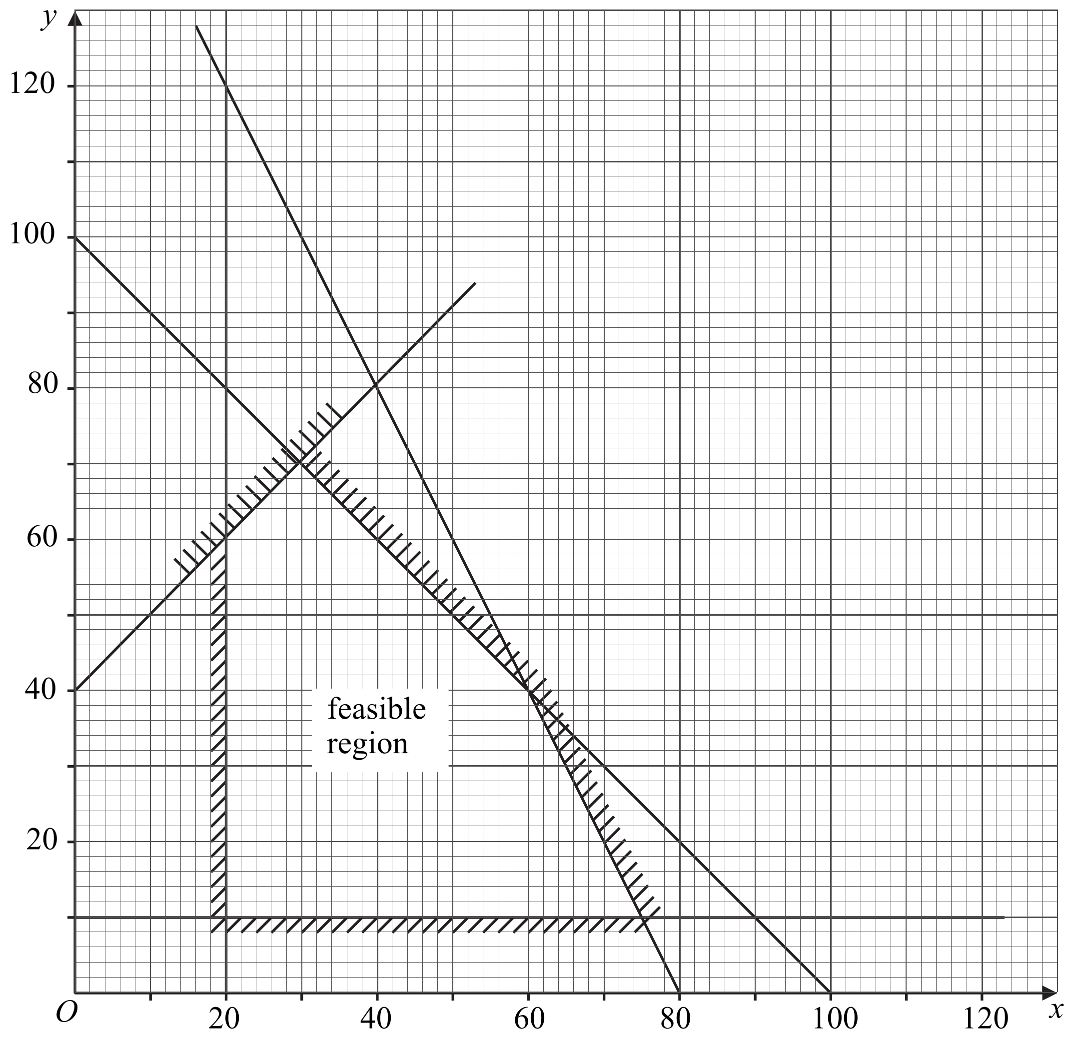


(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your spanning tree. *(1 mark)*

(iii) Draw your spanning tree. *(2 marks)*

4 The diagram shows the feasible region of a linear programming problem.



(a) On the feasible region, find:

(i) the maximum value of $2x + 3y$; *(2 marks)*

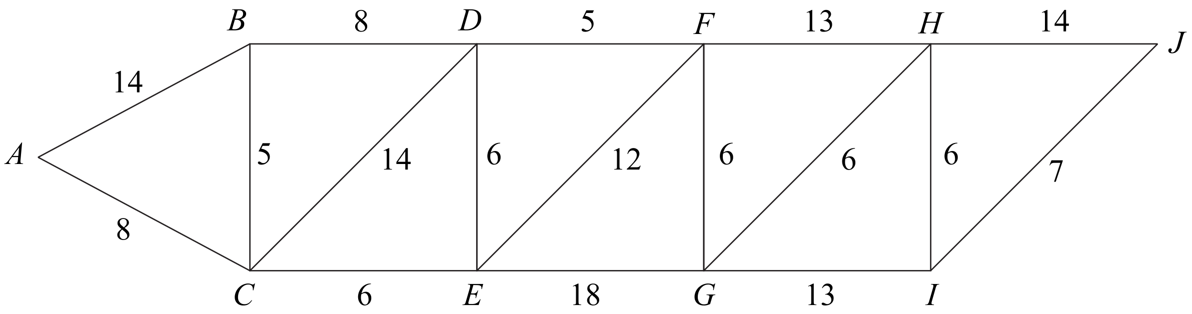
(ii) the maximum value of $3x + 2y$; *(2 marks)*

(iii) the minimum value of $-2x + y$. *(2 marks)*

(b) Find the 5 inequalities that define the feasible region. *(6 marks)*

5 [**Figure 1**, printed on the insert, is provided for use in this question.]

The network shows the times, in minutes, to travel between 10 towns.



(a) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to travel from A to J. *(6 marks)*

(b) State the corresponding route. *(1 mark)*

6 Two algorithms are shown.

Algorithm 1

Line 10	Input P
Line 20	Input R
Line 30	Input T
Line 40	Let $I = (P * R * T)/100$
Line 50	Let $A = P + I$
Line 60	Let $M = A/(12 * T)$
Line 70	Print M
Line 80	Stop

Algorithm 2

Line 10	Input P
Line 20	Input R
Line 30	Input T
Line 40	Let $A = P$
Line 50	$K = 0$
Line 60	Let $K = K + 1$
Line 70	Let $I = (A * R)/100$
Line 80	Let $A = A + I$
Line 90	If $K < T$ then goto Line 60
Line 100	Let $M = A/(12 * T)$
Line 110	Print M
Line 120	Stop

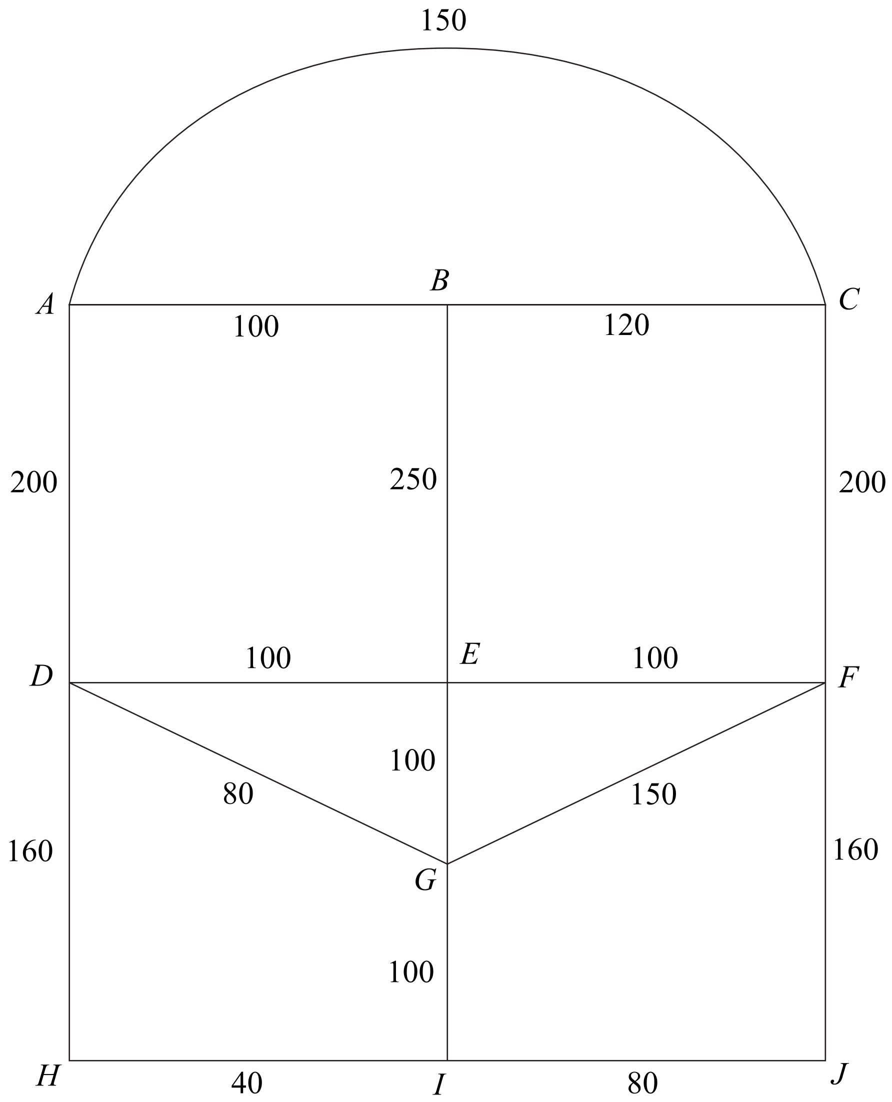
In the case where the input values are $P = 400$, $R = 5$ and $T = 3$:

(a) trace **Algorithm 1**; *(3 marks)*

(b) trace **Algorithm 2**. *(4 marks)*

P80606/Jan06/MD01

7 Stella is visiting Tijuana on a day trip. The diagram shows the lengths, in metres, of the roads near the bus station.



Total = 2090

Stella leaves the bus station at A. She decides to walk along all of the roads at least once before returning to A.

(a) Explain why it is not possible to start from A, travel along each road only once and return to A. *(1 mark)*

(b) Find the length of an optimal 'Chinese postman' route around the network, starting and finishing at A. *(5 marks)*

(c) At each of the 9 places B, C, …, J, there is a statue. Find the number of times that Stella will pass a statue if she follows her optimal route. *(2 marks)*

8 Salvadore is visiting six famous places in Barcelona: La Pedrera (L), Nou Camp (N), Olympic Village (O), Park Guell (P), Ramblas (R) and Sagrada Familia (S). Owing to the traffic system the time taken to travel between two places may vary according to the direction of travel.

The table shows the times, in minutes, that it will take to travel between the six places.

From \ **To**	**La Pedrera** (L)	**Nou Camp** (N)	**Olympic Village** (O)	**Park Guell** (P)	**Ramblas** (R)	**Sagrada Familia** (S)
La Pedrera (L)	—	35	30	30	37	35
Nou Camp (N)	25	—	20	21	25	40
Olympic Village (O)	15	40	—	25	30	29
Park Guell (P)	30	35	25	—	35	20
Ramblas (R)	20	30	17	25	—	25
Sagrada Familia (S)	25	35	29	20	30	—

(a) Find the total travelling time for:

(i) the route $LNOL$; *(1 mark)*

(ii) the route $LONL$. *(1 mark)*

(b) Give an example of a Hamiltonian cycle in the context of the above situation. *(1 mark)*

(c) Salvadore intends to travel from one place to another until he has visited all of the places before returning to his starting place.

(i) Show that, using the nearest neighbour algorithm starting from Sagrada Familia (S), the total travelling time for Salvadore is 145 minutes. *(3 marks)*

(ii) Explain why your answer to part (c)(i) is an upper bound for the minimum travelling time for Salvadore. *(2 marks)*

(iii) Salvadore starts from Sagrada Familia (S) and then visits Ramblas (R). Given that he visits Nou Camp (N) before Park Guell (P), find an improved upper bound for the total travelling time for Salvadore. *(3 marks)*

Turn over for the next question

9 A factory makes three different types of widget: plain, bland and ordinary. Each widget is made using three different machines: A, B and C.

Each plain widget needs 5 minutes on machine A, 12 minutes on machine B and 24 minutes on machine C.

Each bland widget needs 4 minutes on machine A, 8 minutes on machine B and 12 minutes on machine C.

Each ordinary widget needs 3 minutes on machine A, 10 minutes on machine B and 18 minutes on machine C.

Machine A is available for 3 hours a day, machine B for 4 hours a day and machine C for 9 hours a day.

The factory must make:

more plain widgets than bland widgets;

more bland widgets than ordinary widgets.

At least 40% of the total production must be plain widgets.

Each day, the factory makes x plain, y bland and z ordinary widgets.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, writing your answers with simplified integer coefficients. *(8 marks)*

END OF QUESTIONS

Surname		Other Names							
Centre Number						Candidate Number			
Candidate Signature									

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Insert

Wednesday 18 January 2006 1.30 pm to 3.00 pm

Insert for use in **Question 5**.

Fill in the boxes at the top of this page.

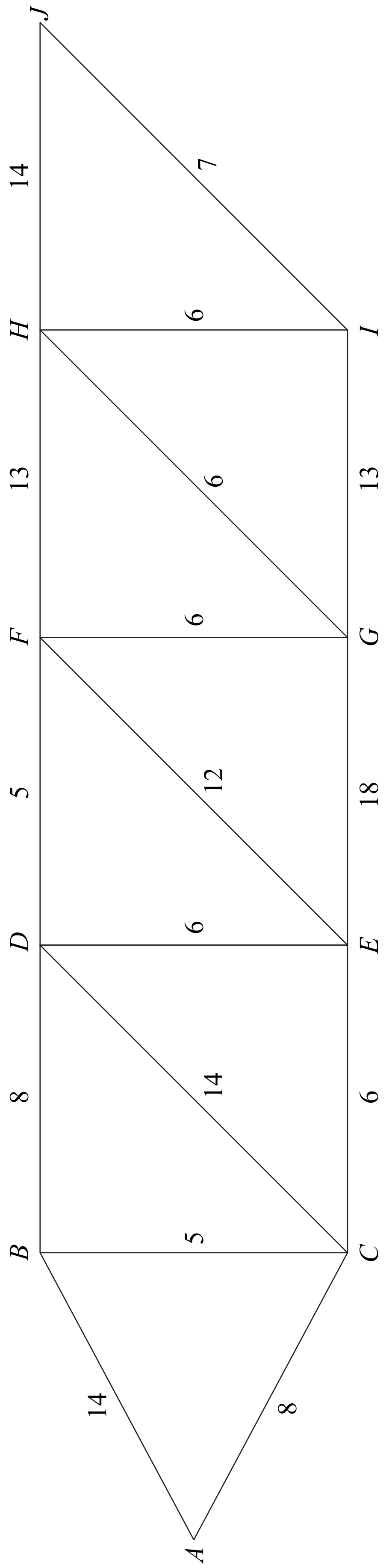
Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

P80606/Jan06/MD01 6/6/6/

Figure 1 (for Question 5)



General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Thursday 8 June 2006 9.00 am to 10.30 am

For this paper you must have:
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- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 3, 5 and 6 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

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Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 Five people, A, B, C, D and E, are to be matched to five tasks, 1, 2, 3, 4 and 5. The table shows which tasks each person can do.

Person	**Tasks**
A	1, 3, 5
B	2, 4
C	2
D	4, 5
E	3, 5

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially A is matched to task 3, B to task 4, C to task 2 and E to task 5.

Use an alternating path from this initial matching to find a complete matching. *(4 marks)*

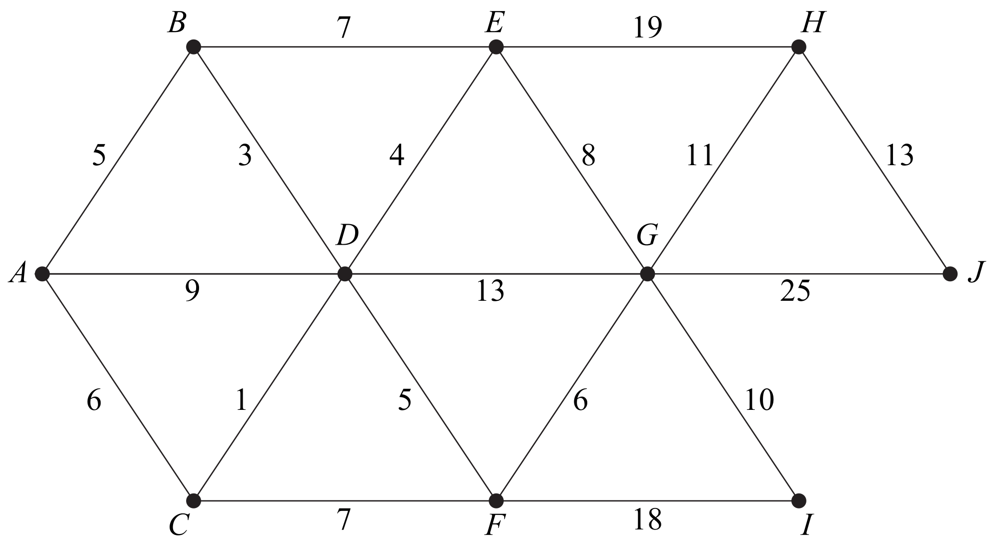
2 (a) Use a shuttle sort to rearrange the following numbers into ascending order.

18 2 12 7 26 19 16 24 *(5 marks)*

(b) State the number of comparisons and swaps (exchanges) for each of the first three passes. *(3 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in **part (b)** of this question.]

The diagram shows a network of roads. The number on each edge is the length, in kilometres, of the road.



(a) (i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

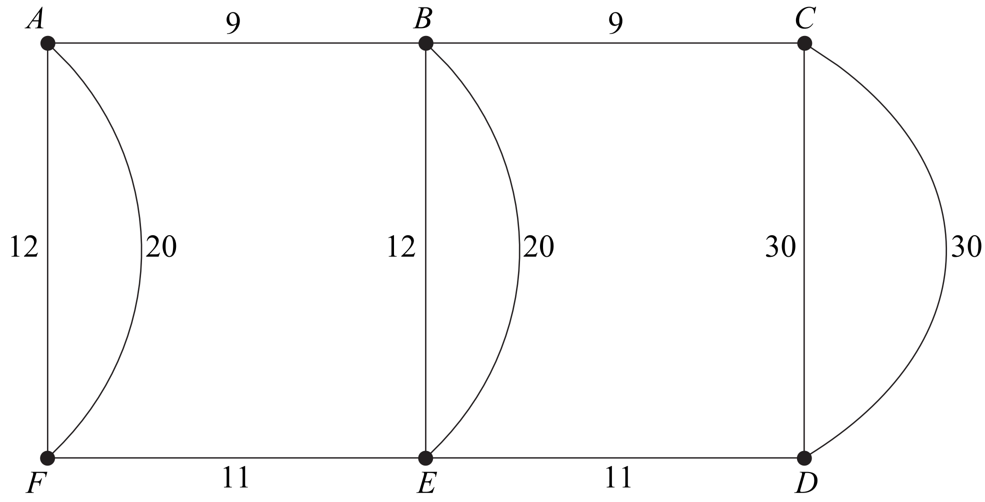
(ii) State the length of your minimum spanning tree. *(1 mark)*

(b) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest distance from A to J. *(6 marks)*

(ii) A new road, of length x km, is built connecting I to J. The minimum distance from A to J is reduced by using this new road. Find, and solve, an inequality for x. *(2 marks)*

Turn over for the next question

4 The diagram shows a network of roads connecting 6 villages. The number on each edge is the length, in miles, of the road.



Total length of the roads = 164 miles

(a) A police patrol car based at village A has to travel along each road at least once before returning to A. Find the length of an optimal 'Chinese postman' route for the police patrol car. *(6 marks)*

(b) A council worker starts from A and travels along each road at least once before finishing at C. Find the length of an optimal route for the council worker. *(2 marks)*

(c) A politician is to travel along all the roads at least once. He can start his journey at any village and can finish his journey at any village.

(i) Find the length of an optimal route for the politician. *(2 marks)*

(ii) State the vertices from which the politician could start in order to achieve this optimal route. *(1 mark)*

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

(a) Gill is solving a travelling salesperson problem.

(i) She finds the following upper bounds: 7.5, 8, 7, 7.5, 8.5 .

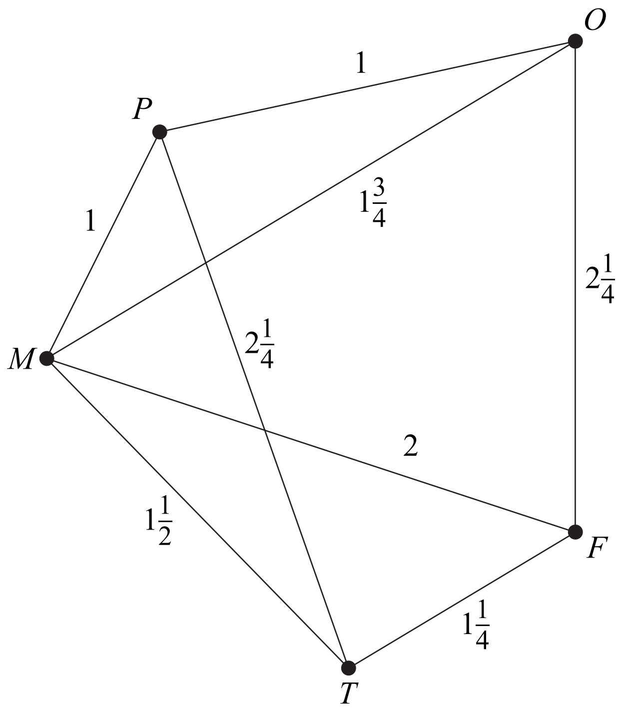
Write down the best upper bound. *(1 mark)*

(ii) She finds the following lower bounds: 6.5, 7, 6.5, 5, 7.

Write down the best lower bound. *(1 mark)*

(b) George is travelling by plane to a number of cities. He is to start at F and visit each of the other cities at least once before returning to F.

The diagram shows the times of flights, in hours, between cities. Where no time is shown, there is no direct flight available.



(i) Complete **Figure 2** to show the minimum times to travel between all pairs of cities. *(2 marks)*

(ii) Find an upper bound for the minimum total flying time by using the route *FTPOMF*. *(1 mark)*

(iii) Using the nearest neighbour algorithm starting from F, find an upper bound for the minimum total flying time. *(4 marks)*

(iv) By deleting F, find a lower bound for the minimum total flying time. *(5 marks)*

6 [**Figure 3**, printed on the insert, is provided for use in this question.]

Ernesto is to plant a garden with two types of tree: palms and conifers.

He is to plant at least 10, but not more than 80 palms.

He is to plant at least 5, but not more than 40 conifers.

He cannot plant more than 100 trees in total.

Each palm needs 20 litres of water each day and each conifer needs 60 litres of water each day. There are 3000 litres of water available each day.

Ernesto makes a profit of £2 on each palm and £1 on each conifer that he plants and he wishes to maximise his profit.

Ernesto plants x palms and y conifers.

(a) Formulate Ernesto's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 3**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(7 marks)*

(c) Find the maximum profit for Ernesto. *(2 marks)*

(d) Ernesto introduces a new pricing structure in which he makes a profit of £1 on each palm and £4 on each conifer.

Find Ernesto's new maximum profit and the number of each type of tree that he should plant to obtain this maximum profit. *(2 marks)*

7 A connected graph **G** has m vertices and n edges.

(a) (i) Write down the number of edges in a minimum spanning tree of **G**. *(1 mark)*

(ii) Hence write down an inequality relating m and n. *(2 marks)*

(b) The graph **G** contains a Hamiltonian cycle. Write down the number of edges in this cycle. *(1 mark)*

(c) In the case where **G** is Eulerian, draw a graph of **G** for which $m = 6$ and $n = 12$. *(2 marks)*

END OF QUESTIONS

Surname		Other Names						
Centre Number						Candidate Number		
Candidate Signature								

General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Insert

Thursday 8 June 2006 9.00 am to 10.30 am

Insert for use in **Questions 3, 5 and 6.**

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 3(b))

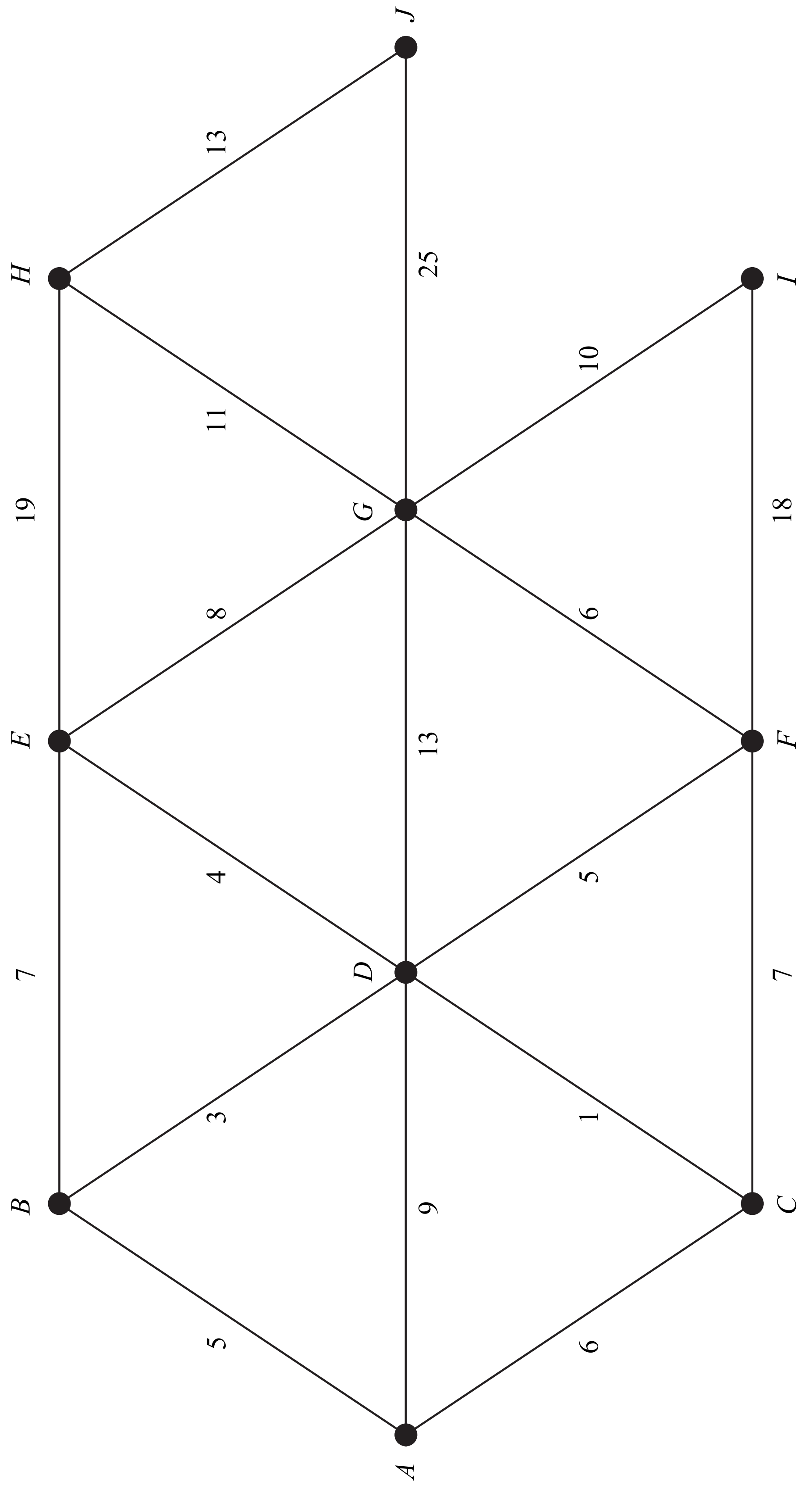
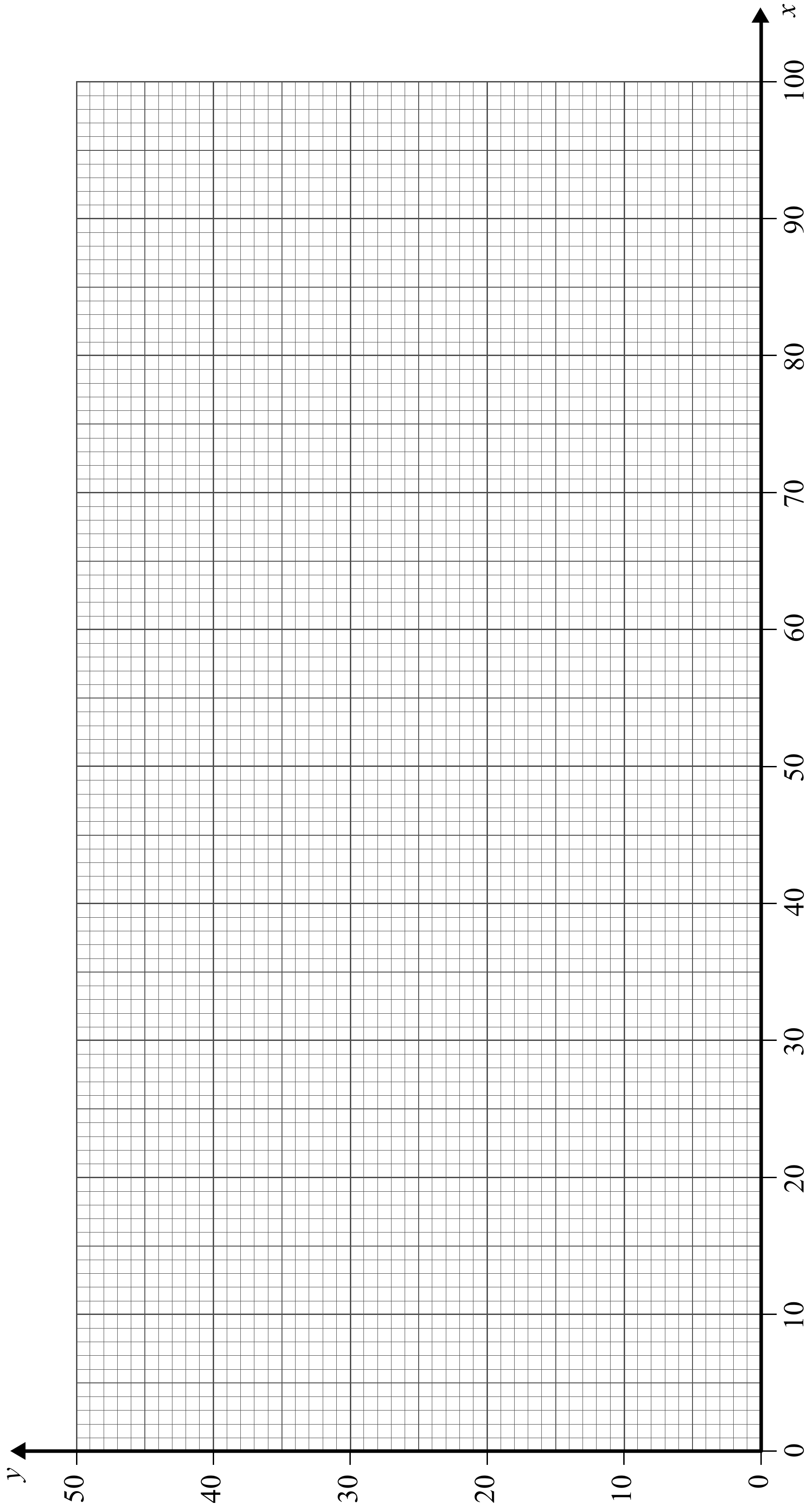


Figure 2 (for use in Question 5)

	M	P	O	T	F
M	–	1	$1\frac{3}{4}$	$1\frac{1}{2}$	2
P	1	–	1	$2\frac{1}{4}$	
O	$1\frac{3}{4}$	1	–		$2\frac{1}{4}$
T	$1\frac{1}{2}$	$2\frac{1}{4}$		–	$1\frac{1}{4}$
F	2		$2\frac{1}{4}$	$1\frac{1}{4}$	–

Turn over ▶

Figure 3 (for use in Question 6)



General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Tuesday 16 January 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 6 and 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
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Information
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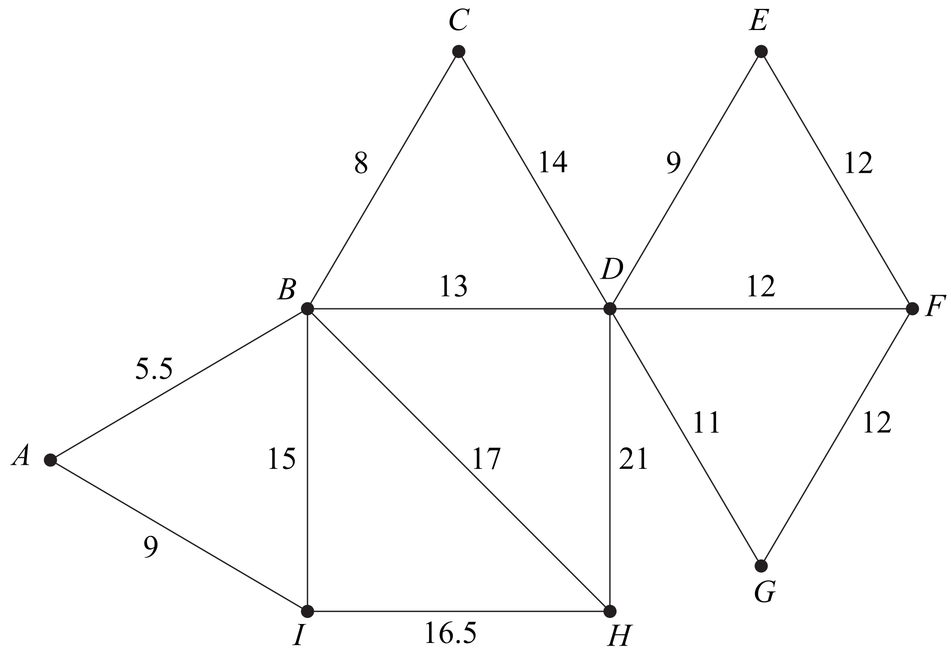
P91441/Jan07/MD01 6/6/6/

Answer **all** questions.

1 The following network shows the lengths, in miles, of roads connecting nine villages.



(a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(b) Find the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(d) State the number of other spanning trees that are of the same length as your answer in part (a). *(1 mark)*

2 Five people A, B, C, D and E are to be matched to five tasks R, S, T, U and V.

The table shows the tasks that each person is able to undertake.

Person	**Tasks**
A	R, V
B	R, T
C	T, V
D	U, V
E	S, U

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task V, B to task R, C to task T, and E to task U.

Demonstrate, by using an alternating path from this initial matching, how each person can be matched to a task. *(4 marks)*

3 Mark is driving around the one-way system in Leicester. The following table shows the times, in minutes, for Mark to drive between four places: A, B, C and D. Mark decides to start from A, drive to the other three places and then return to A.

Mark wants to keep his driving time to a minimum.

From \ **To**	***A***	***B***	***C***	***D***
A	–	8	6	11
B	14	–	13	25
C	14	9	–	17
D	26	10	18	–

(a) Find the length of the tour $ABCDA$. *(2 marks)*

(b) Find the length of the tour $ADCBA$. *(1 mark)*

(c) Find the length of the tour using the nearest neighbour algorithm starting from A. *(4 marks)*

(d) Write down which of your answers to parts (a), (b) and (c) gives the best upper bound for Mark's driving time. *(1 mark)*

Turn over ▶

4 (a) A student is using a bubble sort to rearrange seven numbers into ascending order.

Her correct solution is as follows:

Initial list	18	17	13	26	10	14	24
After 1st pass	17	13	18	10	14	24	26
After 2nd pass	13	17	10	14	18	24	26
After 3rd pass	13	10	14	17	18	24	26
After 4th pass	10	13	14	17	18	24	26
After 5th pass	10	13	14	17	18	24	26

Write down the number of comparisons and swaps on each of the five passes.
(6 marks)

(b) Find the maximum number of comparisons and the maximum number of swaps that might be needed in a bubble sort to rearrange seven numbers into ascending order.
(2 marks)

5 A student is using the following algorithm with different values of A and B.

Line 10	Input A, B
Line 20	Let $C = 0$ and let $D = 0$
Line 30	Let $C = C + A$
Line 40	Let $D = D + B$
Line 50	If $C = D$ then go to Line 110
Line 60	If $C > D$ then go to Line 90
Line 70	Let $C = C + A$
Line 80	Go to Line 50
Line 90	Let $D = D + B$
Line 100	Go to Line 50
Line 110	Print C
Line 120	End

(a) (i) Trace the algorithm in the case where $A = 2$ and $B = 3$. *(3 marks)*

(ii) Trace the algorithm in the case where $A = 6$ and $B = 8$. *(3 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

(c) Write down the final value of C in the case where $A = 200$ and $B = 300$. *(1 mark)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

Dino is to have a rectangular swimming pool at his villa.

He wants its width to be at least 2 metres and its length to be at least 5 metres.

He wants its length to be at least twice its width.

He wants its length to be no more than three times its width.

Each metre of the width of the pool costs £1000 and each metre of the length of the pool costs £500.

He has £9000 available.

Let the width of the pool be x metres and the length of the pool be y metres.

(a) Show that one of the constraints leads to the inequality

$$2x + y \leqslant 18$$ *(1 mark)*

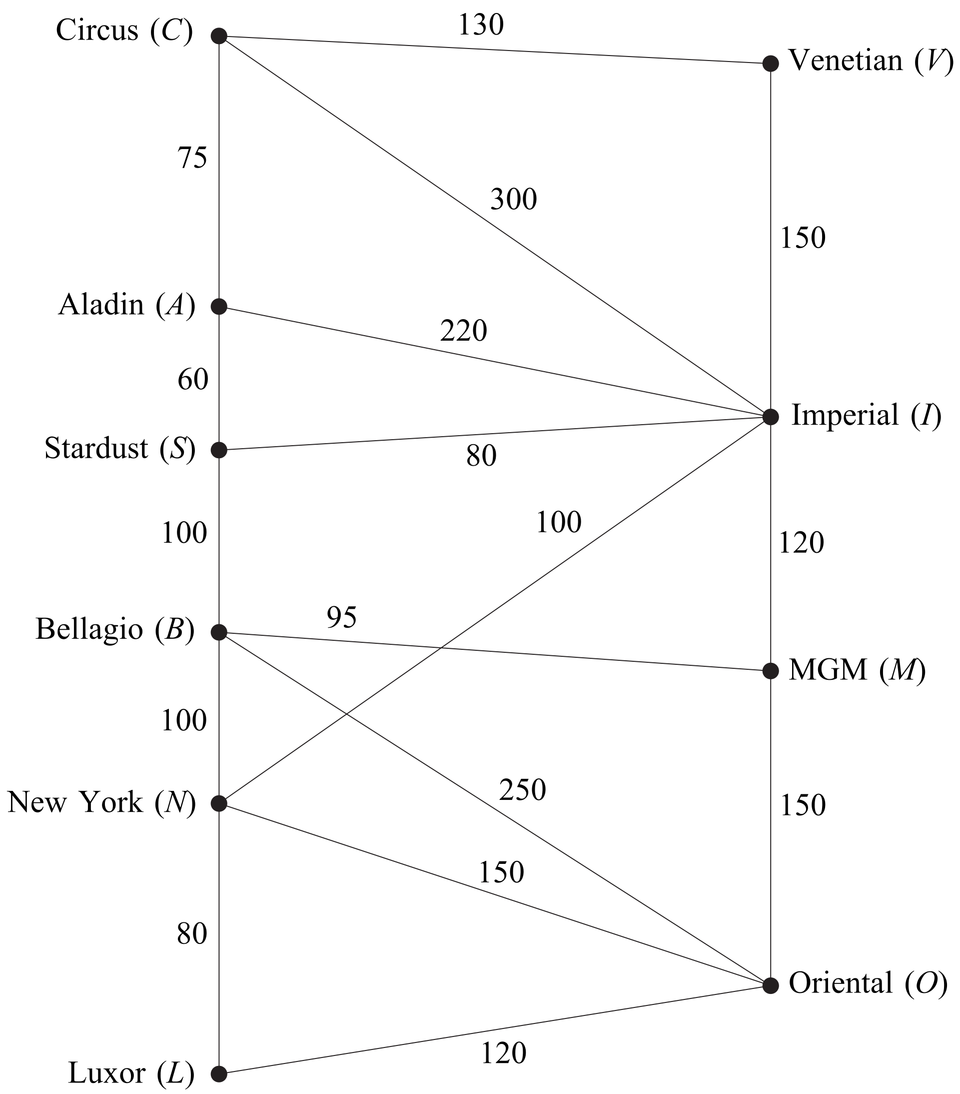
(b) Find four further inequalities. *(3 marks)*

(c) On **Figure 1**, draw a suitable diagram to show the feasible region. *(6 marks)*

(d) Use your diagram to find the maximum width of the pool. State the corresponding length of the pool. *(3 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows the times, in seconds, taken by Craig to walk along walkways connecting ten hotels in Las Vegas.



The total of all the times in the diagram is 2280 seconds.

(a) (i) Craig is staying at the Circus (C) and has to visit the Oriental (O).

Use Dijkstra's algorithm on **Figure 2** to find the minimum time to walk from C to O. *(6 marks)*

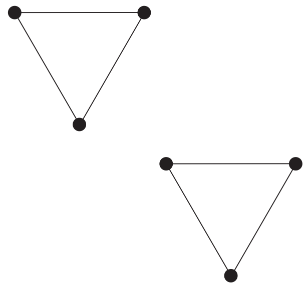
(ii) Write down the corresponding route. *(1 mark)*

(b) (i) Find, by inspection, the shortest time to walk from A to M. *(1 mark)*

(ii) Craig intends to walk along all the walkways. Find the minimum time for Craig to walk along every walkway and return to his starting point. *(6 marks)*

Turn over for the next question

8 (a) The diagram shows a graph **G** with 9 vertices and 9 edges.



(i) State the minimum number of edges that need to be added to **G** to make a connected graph. Draw an example of such a graph. *(2 marks)*

(ii) State the minimum number of edges that need to be added to **G** to make the graph Hamiltonian. Draw an example of such a graph. *(2 marks)*

(iii) State the minimum number of edges that need to be added to **G** to make the graph Eulerian. Draw an example of such a graph. *(2 marks)*

(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

(ii) In addition, the number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(1 mark)*

END OF QUESTIONS

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Centre Number						Candidate Number		
Candidate Signature								

General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Insert

Insert for use in **Questions 6 and 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 6)

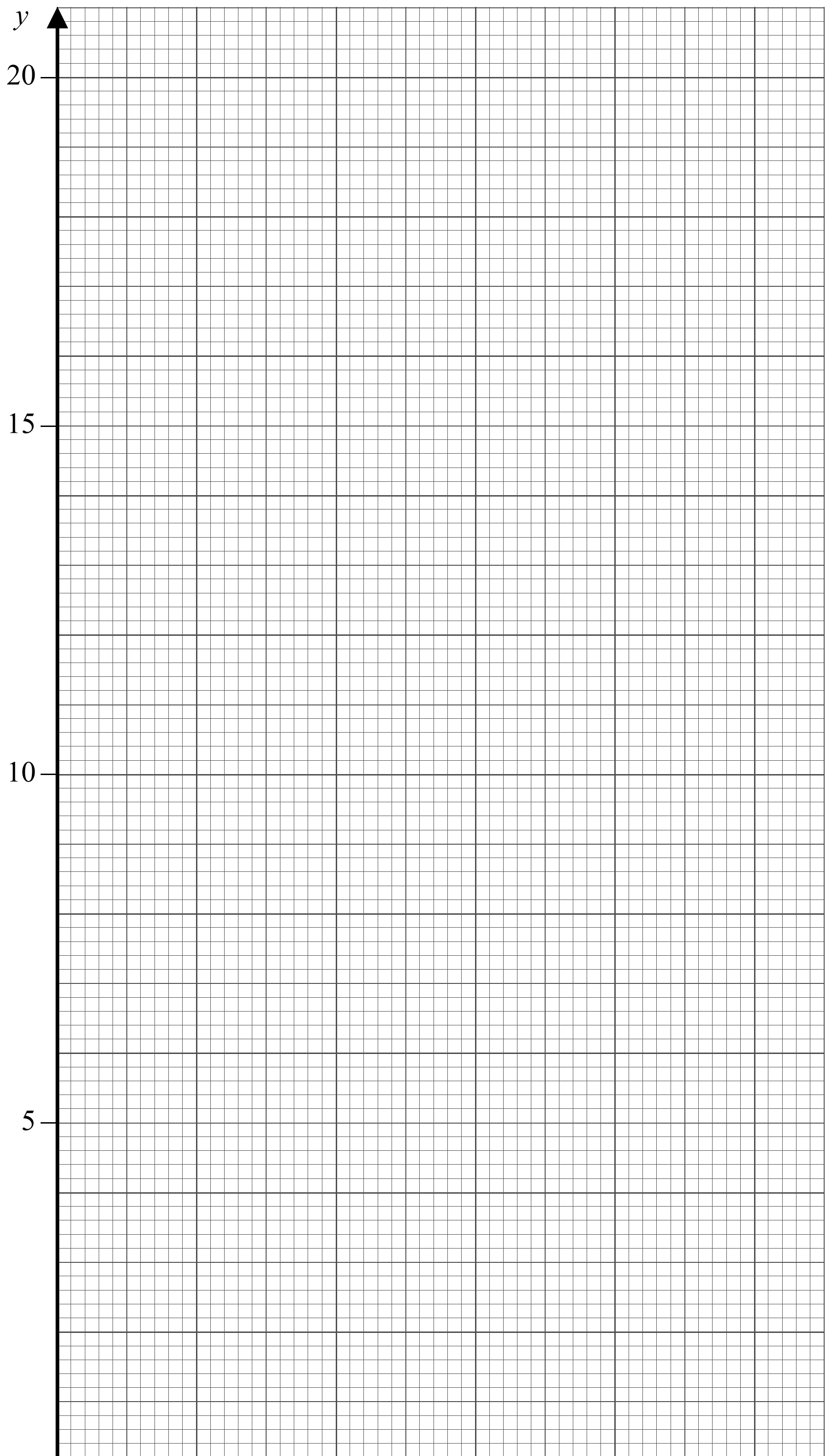
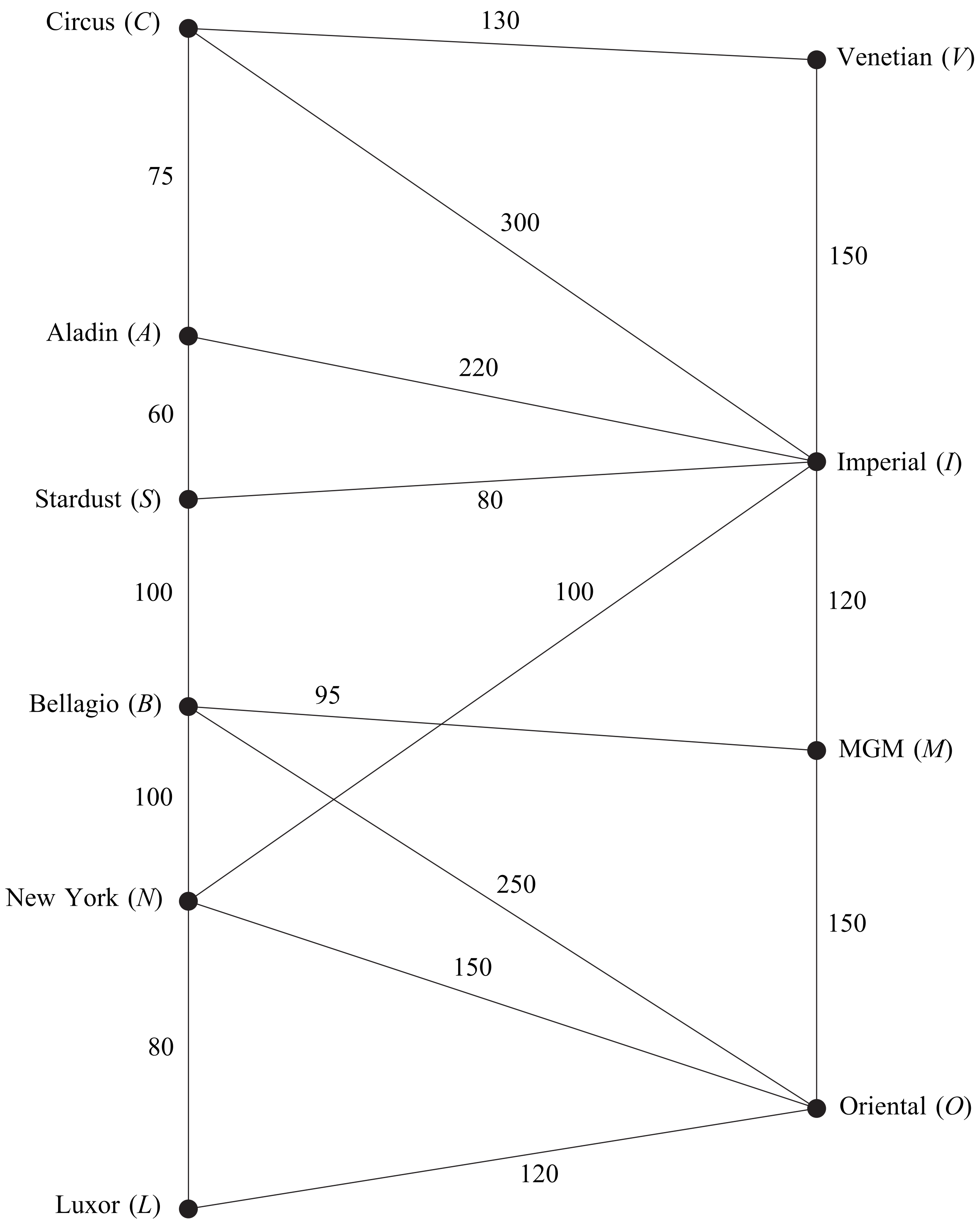


Figure 2 (for use in Question 7)



General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

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- The marks for questions are shown in brackets.

Answer **all** questions.

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**	**Task 6**
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	0	1	0	1	1
D	0	0	0	1	0	0
E	0	1	0	0	0	1
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) At first F insists on being matched to task 4. Explain why, in this case, a complete matching is impossible. *(1 mark)*

(c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*

2 (a) Use a Shell sort to rearrange the following numbers into ascending order, showing the new arrangement after each pass.

28 22 20 17 14 11 6 5 *(5 marks)*

(b) (i) Write down the number of comparisons on the first pass. *(1 mark)*

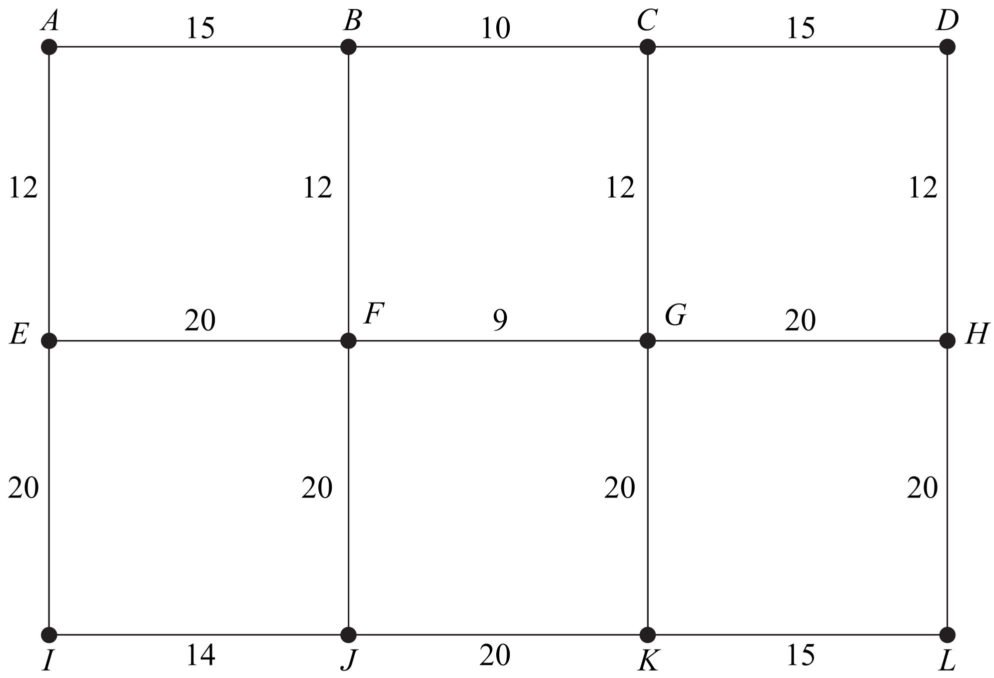
(ii) Write down the number of swaps on the first pass. *(1 mark)*

(c) Find the total number of comparisons needed to rearrange the original list of 8 numbers into ascending order using a shuttle sort.

(You do not need to perform a shuttle sort.) *(1 mark)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The following network represents the footpaths connecting 12 buildings on a university campus. The number on each edge represents the time taken, in minutes, to walk along a footpath.



(a) (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from A to L. *(7 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) A new footpath is to be constructed. There are two possibilities:

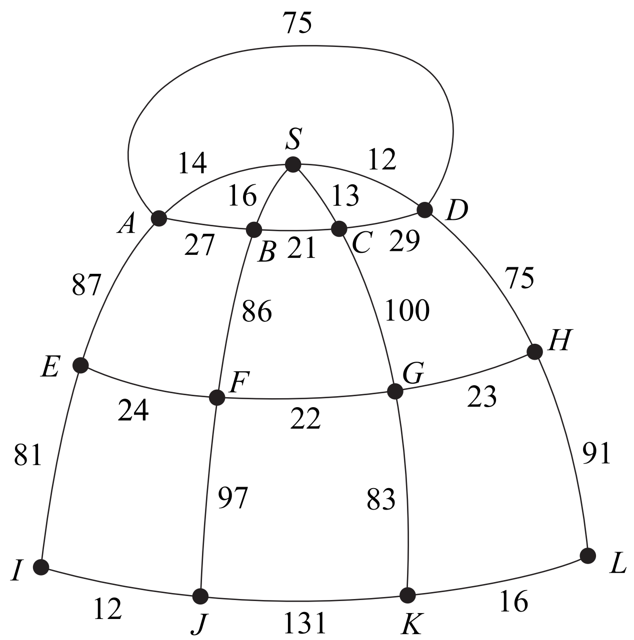
from A to D, with a walking time of 30 minutes; or

from A to I, with a walking time of 20 minutes.

Determine which of the two alternative new footpaths would reduce the walking time from A to L by the greater amount. *(3 marks)*

4 The diagram shows the various ski-runs at a ski resort. There is a shop at S. The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points A, B,...,L and at the shop at S.

The number on each edge represents the distance, in metres, between two points.



Total of all edges $= 1135$

(a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points A, B,...,L and the shop at S.

(i) Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(3 marks)*

(iv) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree. *(2 marks)*

(b) At the end of each day a snow plough has to drive at least once along each edge shown in the diagram in preparation for the following day's skiing. The snow plough must start and finish at the point L.

Use the Chinese Postman algorithm to find the minimum distance that the snow plough must travel. *(6 marks)*

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

The Jolly Company sells two types of party pack: excellent and luxury.

Each excellent pack has five balloons and each luxury pack has ten balloons.

Each excellent pack has 32 sweets and each luxury pack has 8 sweets.

The company has 1500 balloons and 4000 sweets available.

The company sells at least 50 of each type of pack and at least 140 packs in total.

The company sells x excellent packs and y luxury packs.

(a) Show that the above information can be modelled by the following inequalities.

$$x + 2y \leqslant 300, \quad 4x + y \leqslant 500, \quad x \geqslant 50, \quad y \geqslant 50, \quad x + y \geqslant 140$$ *(4 marks)*

(b) The company sells each excellent pack for 80p and each luxury pack for £1.20. The company needs to find its minimum and maximum total income.

(i) On **Figure 2**, draw a suitable diagram to enable this linear programming problem to be solved graphically, indicating the feasible region and an objective line. *(8 marks)*

(ii) Find the company's maximum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

(iii) Find the company's minimum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

Turn over for the next question

6 (a) Mark is staying at the Grand Hotel (G) in Oslo. He is going to visit four famous places in Oslo: Aker Brygge (A), the National Theatre (N), Parliament House (P) and the Royal Palace (R).

The figures in the table represent the walking times, in seconds, between the places.

	Grand Hotel (G)	**Aker Brygge (A)**	**National Theatre (N)**	**Parliament House (P)**	**Royal Palace (R)**
Grand Hotel (G)	–	165	185	65	160
Aker Brygge (A)	165	–	155	115	275
National Theatre (N)	185	155	–	205	125
Parliament House (P)	65	115	205	–	225
Royal Palace (R)	160	275	125	225	–

Mark is to start his tour from the Grand Hotel, visiting each place once before returning to the Grand Hotel. Mark wishes to keep his walking time to a minimum.

(i) Use the nearest neighbour algorithm, starting from the Grand Hotel, to find an upper bound for the walking time for Mark's tour. *(4 marks)*

(ii) By deleting the Grand Hotel, find a lower bound for the walking time for Mark's tour. *(5 marks)*

(iii) The walking time for an optimal tour is T seconds. Use your answers to parts (a)(i) and (a)(ii) to write down a conclusion about T. *(1 mark)*

(b) Mark then intends to start from the Grand Hotel (G), visit three museums, Ibsen (I), Munch (M) and Viking (V), and return to the Grand Hotel. He uses public transport. The table shows the minimum travelling times, in minutes, between the places.

From \ To	**Grand Hotel (G)**	**Ibsen (I)**	**Munch (M)**	**Viking (V)**
Grand Hotel (G)	–	20	17	30
Ibsen (I)	15	–	32	16
Munch (M)	26	18	–	21
Viking (V)	19	27	24	–

(i) Find the length of the tour $GIMVG$. *(1 mark)*

(ii) Find the length of the tour $GVMIG$. *(1 mark)*

(iii) Find the number of different possible tours for Mark. *(1 mark)*

(iv) Write down the number of different possible tours for Mark if he were to visit n museums, starting and finishing at the Grand Hotel. *(1 mark)*

END OF QUESTIONS

Surname		Other Names								
Centre Number						Candidate Number				
Candidate Signature										

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Insert

Insert for use in **Questions 3 and 5**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 3)

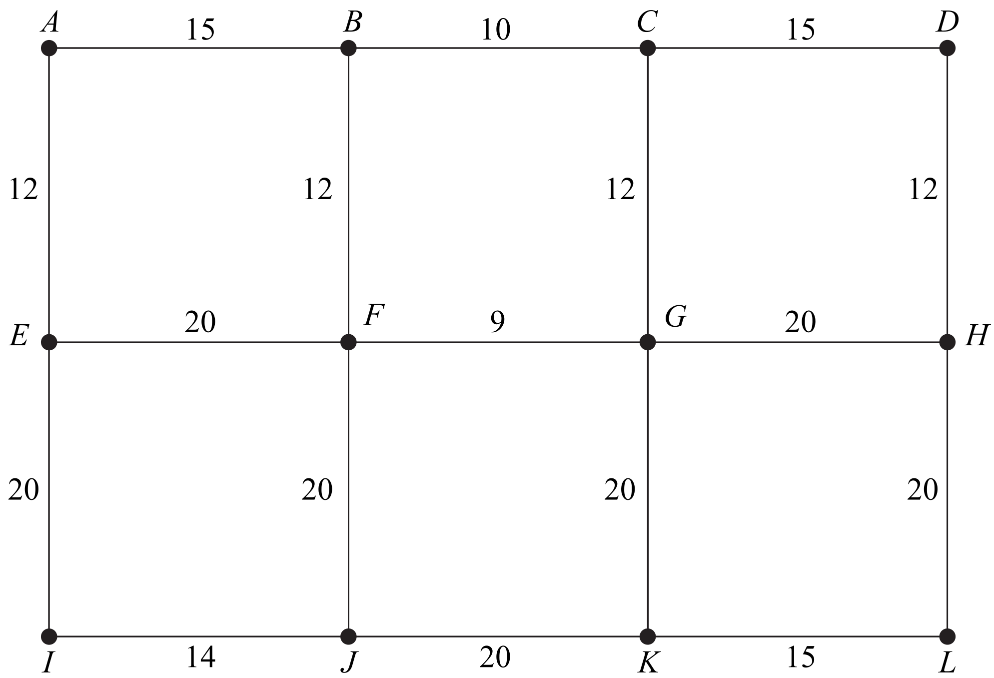
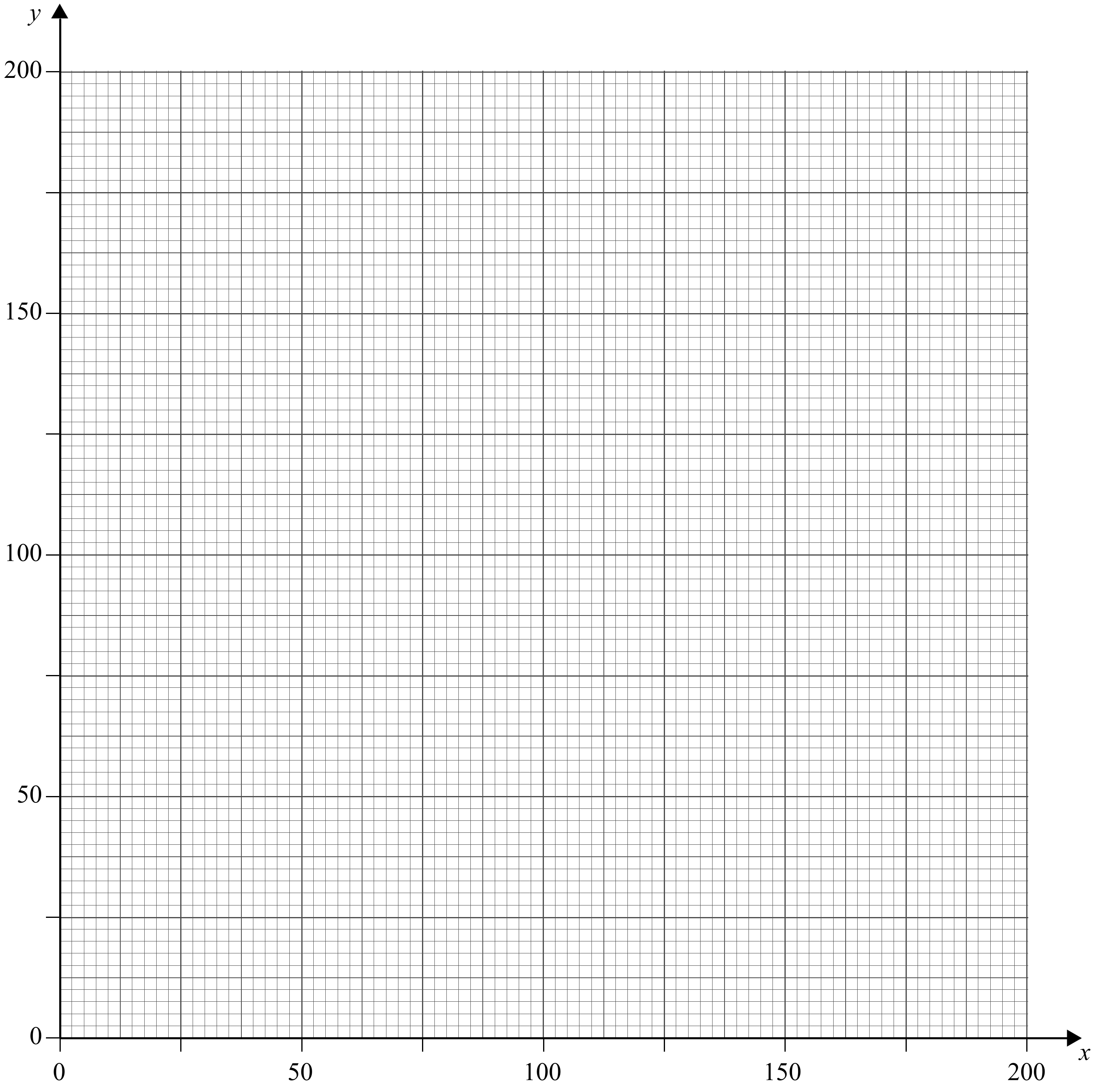


Figure 2 (for use in Question 5)



General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Tuesday 15 January 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 2, 4 and 5 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 Five people, A, B, C, D and E, are to be matched to five tasks, J, K, L, M and N. The table shows the tasks that each person is able to undertake.

Person	**Task**
A	J, N
B	J, L
C	L, N
D	M, N
E	K, M

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task N, B to task J, C to task L, and E to task M.

Complete the alternating path $D-M$ ..., from this initial matching, to demonstrate how each person can be matched to a task. *(3 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by

$$x + y \leqslant 30$$
$$2x + y \leqslant 40$$
$$y \geqslant 5$$
$$x \geqslant 4$$
$$y \geqslant \tfrac{1}{2}x$$

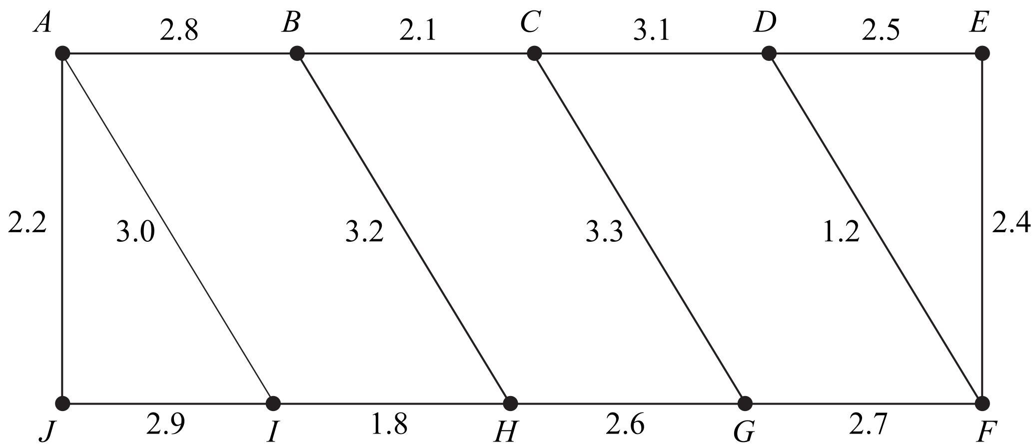
(a) On **Figure 1**, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) Use your diagram to find the maximum value of F, on the feasible region, in the case where:

(i) $F = 3x + y$; *(2 marks)*

(ii) $F = x + 3y$. *(2 marks)*

3 The diagram shows 10 bus stops, A, B, C, ..., J, in Geneva. The number on each edge represents the distance, in kilometres, between adjacent bus stops.



The city council is to connect these bus stops to a computer system which will display waiting times for buses at each of the 10 stops. Cabling is to be laid between some of the bus stops.

(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 bus stops. *(5 marks)*

(b) State the minimum length of cabling needed. *(1 mark)*

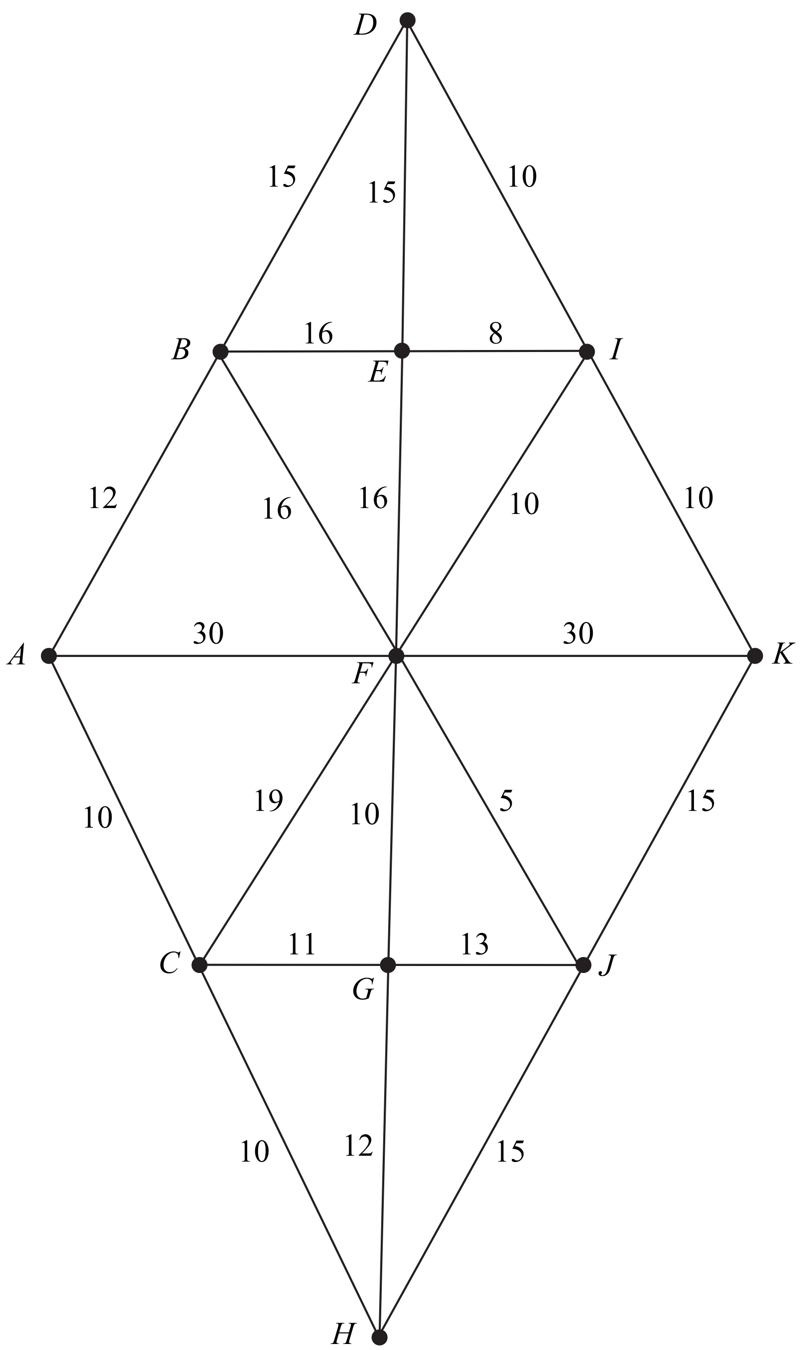
(c) Draw your minimum spanning tree. *(2 marks)*

(d) If Prim's algorithm, starting from A, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(2 marks)*

Turn over for the next question

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows 11 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



The total of all of the times is 308 minutes.

(a) (i) Use Dijkstra's algorithm on **Figure 2** to find the minimum time to travel from A to K. *(6 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) Find the length of an optimum Chinese postman route around the network, starting and finishing at A. (The minimum time to travel from D to H is 40 minutes.) *(5 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

(a) James is solving a travelling salesperson problem.

(i) He finds the following upper bounds: 43, 40, 43, 41, 55, 43, 43.

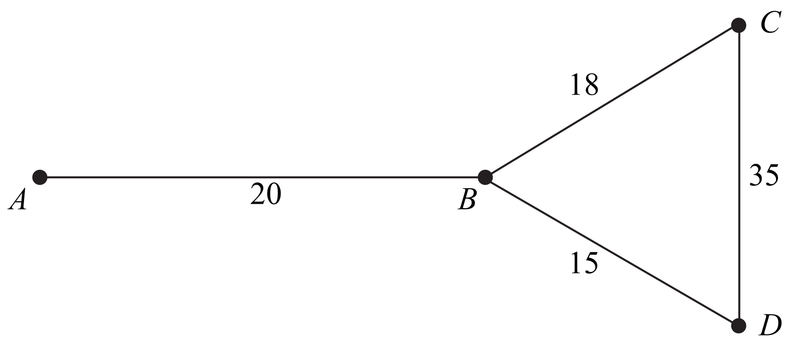
Write down the best upper bound. *(1 mark)*

(ii) James finds the following lower bounds: 33, 40, 33, 38, 33, 38, 38.

Write down the best lower bound. *(1 mark)*

(b) Karen is solving a different travelling salesperson problem and finds an upper bound of 55 and a lower bound of 45. Write down an interpretation of these results. *(1 mark)*

(c) The diagram below shows roads connecting 4 towns, A, B, C and D. The numbers on the edges represent the lengths of the roads, in kilometres, between adjacent towns.



Xiong lives at town A and is to visit each of the other three towns before returning to town A. She wishes to find a route that will minimise her travelling distance.

(i) Complete **Figure 3**, on the insert, to show the shortest distances, in kilometres, between **all** pairs of towns. *(2 marks)*

(ii) Use the nearest neighbour algorithm on **Figure 3** to find an upper bound for the minimum length of a tour of this network that starts and finishes at A. *(3 marks)*

(iii) Hence find the actual route that Xiong would take in order to achieve a tour of the same length as that found in part (c)(ii). *(2 marks)*

6 A student is solving cubic equations that have three different positive integer solutions.

The algorithm that the student is using is as follows:

Line 10	Input A, B, C, D
Line 20	Let $K = 1$
Line 30	Let $N = 0$
Line 40	Let $X = K$
Line 50	Let $Y = AX^3 + BX^2 + CX + D$
Line 60	If $Y \neq 0$ then go to Line 100
Line 70	Print X, "is a solution"
Line 80	Let $N = N + 1$
Line 90	If $N = 3$ then go to Line 120
Line 100	Let $K = K + 1$
Line 110	Go to Line 40
Line 120	End

(a) Trace the algorithm in the case where the input values are:

(i) $A = 1$, $B = -6$, $C = 11$ and $D = -6$; *(4 marks)*

(ii) $A = 1$, $B = -10$, $C = 29$ and $D = -20$. *(4 marks)*

(b) Explain where and why this algorithm will fail if $A = 0$. *(2 marks)*

7 The numbers 17, 3, 16 and 4 are to be sorted into ascending order.

The following four methods are to be compared: bubble sort, shuttle sort, Shell sort and quick sort (with the first number used as the pivot).

A student uses each of the four methods and produces the correct solutions below. Each solution shows the order of the numbers after each pass.

Solution 1	17	3	16	4
	3	17	16	4
	3	16	17	4
	3	4	16	17
Solution 2	17	3	16	4
	16	3	17	4
	3	4	16	17
Solution 3	17	3	16	4
	3	16	4	17
	3	16	4	17
	3	4	16	17
Solution 4	17	3	16	4
	3	16	4	17
	3	4	16	17
	3	4	16	17

(a) Write down which of the four solutions is the bubble sort, the shuttle sort, the Shell sort and the quick sort. *(3 marks)*

(b) For each of the four solutions, write down the number of comparisons and swaps (exchanges) on the first pass. *(8 marks)*

Turn over for the next question

8 Each day, a factory makes three types of hinge: basic, standard and luxury. The hinges produced need three different components: type A, type B and type C.

Basic hinges need 2 components of type A, 3 components of type B and 1 component of type C.

Standard hinges need 4 components of type A, 2 components of type B and 3 components of type C.

Luxury hinges need 3 components of type A, 4 components of type B and 5 components of type C.

Each day, there are 360 components of type A available, 270 of type B and 450 of type C.

Each day, the factory must use at least 720 components in total.

Each day, the factory must use at least 40% of the total components as type A.

Each day, the factory makes x basic hinges, y standard hinges and z luxury hinges.

In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, find five inequalities, each involving x, y and z, which must be satisfied. Simplify each inequality where possible. *(8 marks)*

END OF QUESTIONS

Surname		Other Names							
Centre Number						Candidate Number			
Candidate Signature									

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS
Unit Decision 1

MD01

Insert

Insert for use in **Questions 2, 4 and 5**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 2)

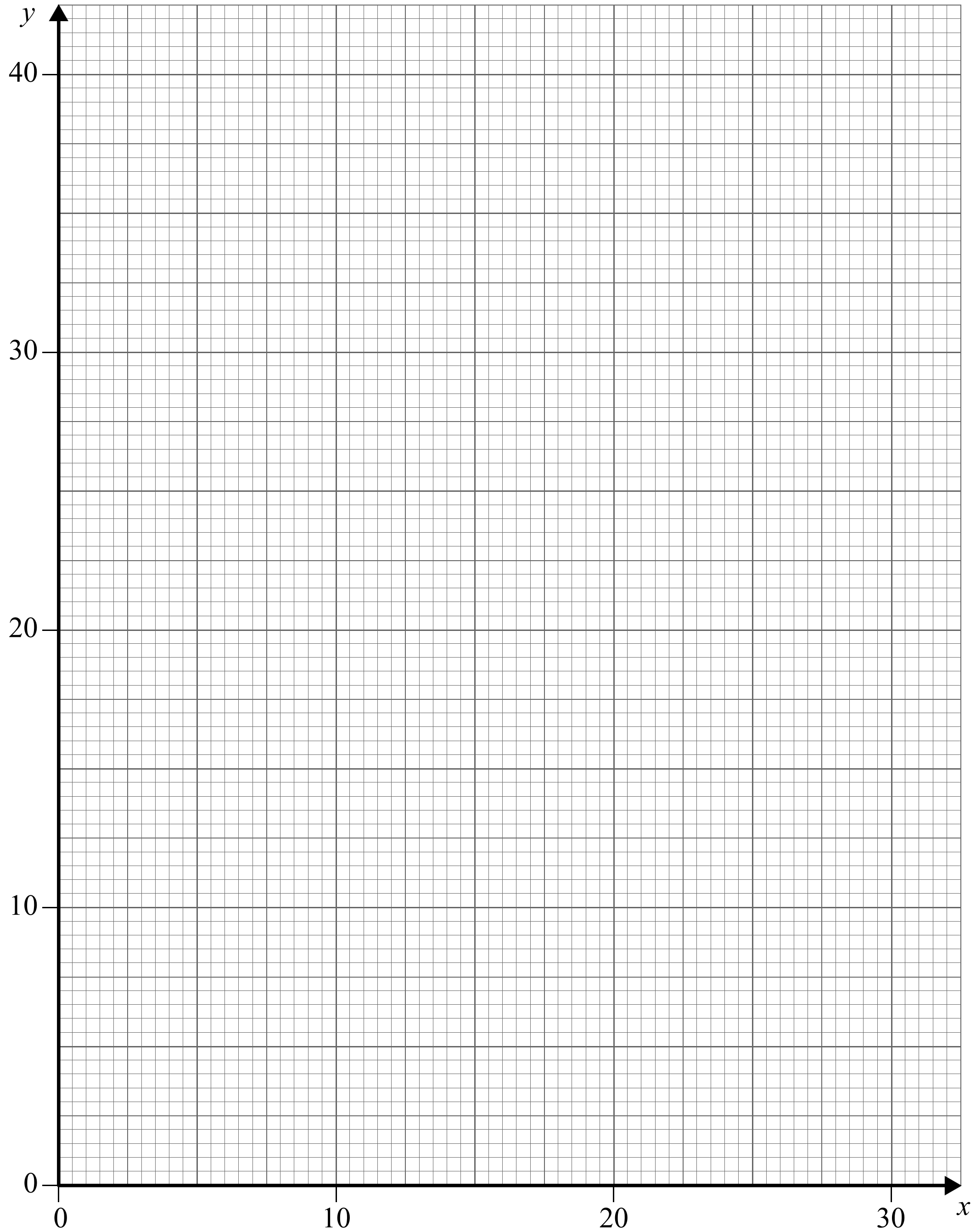
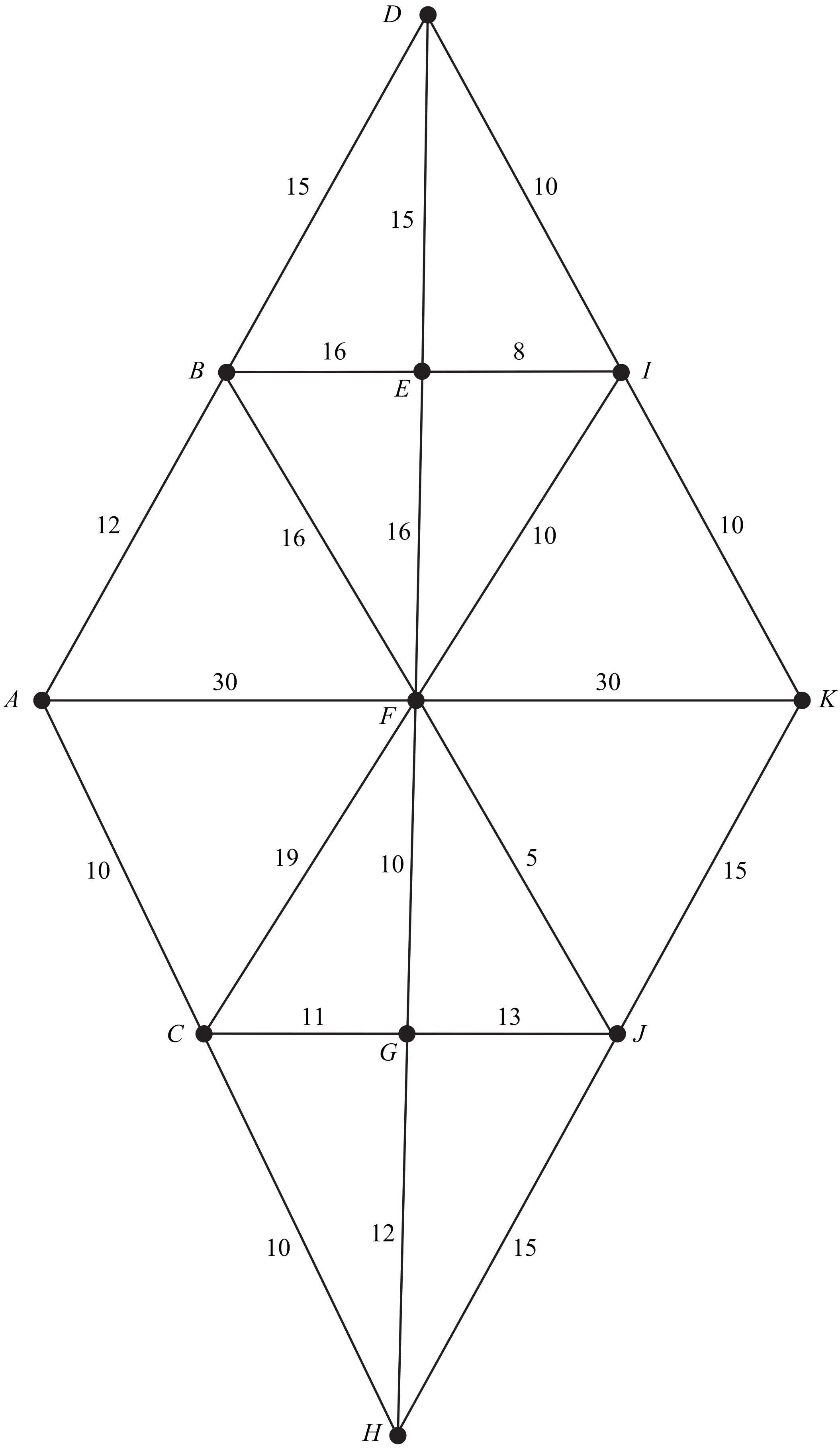


Figure 2 (for use in Question 4)



Turn over ▶

Figure 3 (for use in Question 5)

	A	***B***	***C***	***D***
A	—		38	
B		—		
C	38		—	
D				—

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Friday 6 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 6 and 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6.

The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**	**Task 6**
A	0	0	1	0	1	1
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	1	0	1	0	1	0
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task 3, B to task 4, C to task 2 and E to task 5. From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(5 marks)*

2 (a) Use a quick sort to rearrange the following letters into alphabetical order. You must indicate the pivot that you use at each pass.

P B M N J K R D *(5 marks)*

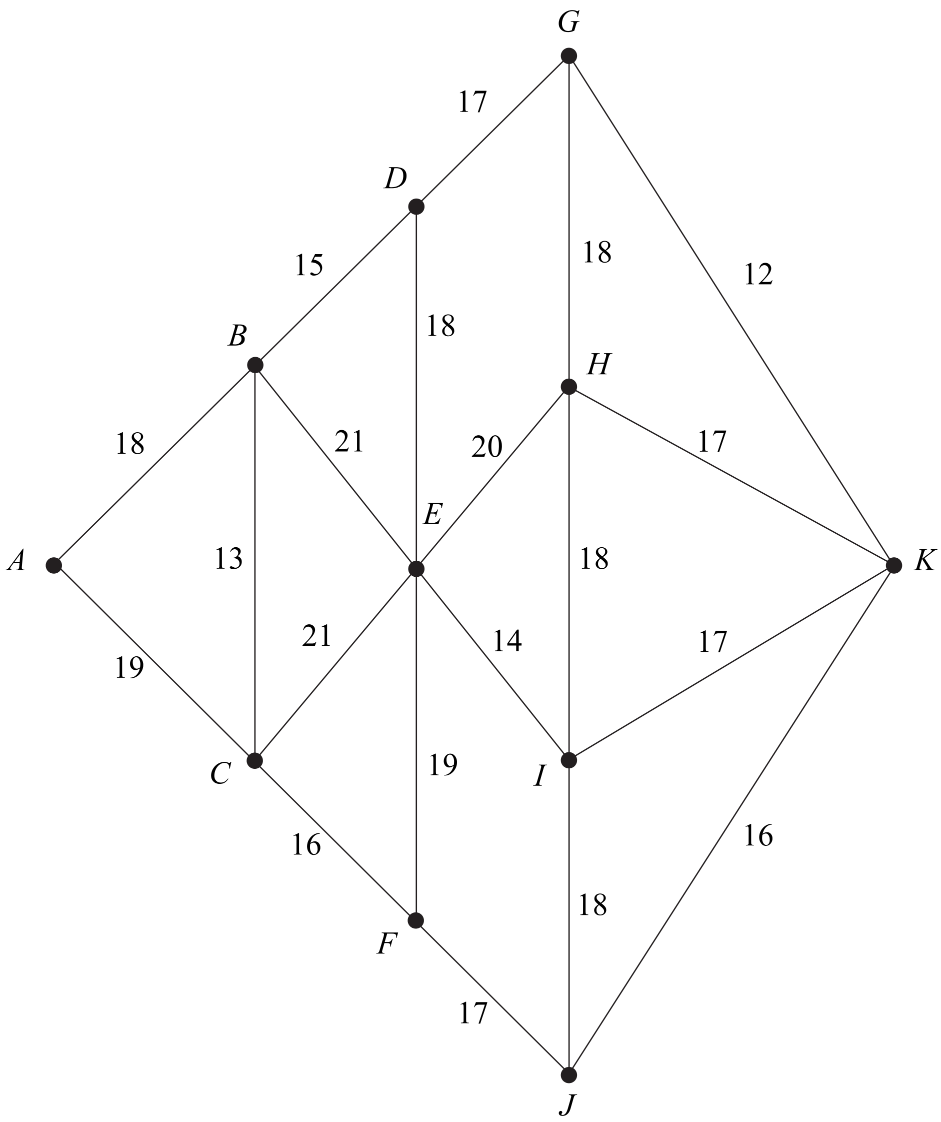
(b) (i) Find the maximum number of swaps needed to rearrange a list of 8 numbers into ascending order when using a **bubble** sort. *(1 mark)*

(ii) A list of 8 numbers was rearranged into ascending order using a **bubble** sort. The maximum number of swaps was needed. What can be deduced about the original list of numbers? *(1 mark)*

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 11 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 11 vertices, A, B, ..., K. The number on each edge represents the distance, in miles, between a pair of vertices.



(i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(ii) Find the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

4 David, a tourist, wishes to visit five places in Rome: Basilica (B), Coliseum (C), Pantheon (P), Trevi Fountain (T) and Vatican (V). He is to start his tour at one of the places, visit each of the other places, before returning to his starting place.

The table shows the times, in minutes, to travel between these places. David wishes to keep his travelling time to a minimum.

	B	***C***	***P***	***T***	***V***
B	–	43	57	52	18
C	43	–	18	13	56
P	57	18	–	8	48
T	52	13	8	–	51
V	18	56	48	51	–

(a) (i) Find the total travelling time for the tour $TPVBCT$. *(1 mark)*

(ii) Find the total travelling time for David's tour using the nearest neighbour algorithm starting from T. *(4 marks)*

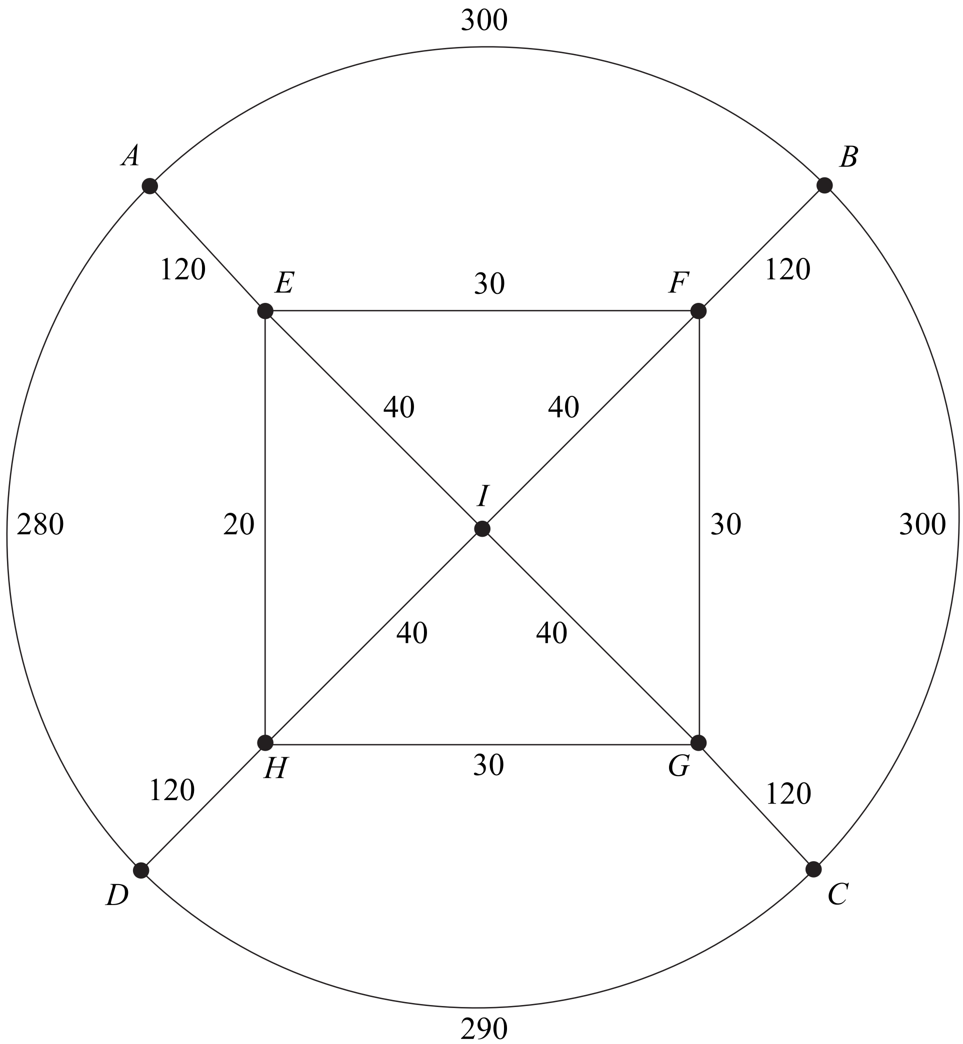
(iii) Explain why your answer to part (a)(ii) is an upper bound for David's minimum total travelling time. *(2 marks)*

(b) (i) By deleting B, find a lower bound for the total travelling time for the minimum tour. *(5 marks)*

(ii) Explain why your answer to part (b)(i) is a lower bound for David's minimum total travelling time. *(2 marks)*

(c) Sketch a network showing the edges that give the lower bound found in part (b)(i) and comment on its significance. *(2 marks)*

5 The diagram shows a network of sixteen roads on a housing estate. The number on each edge is the length, in metres, of the road. The total length of the sixteen roads is 1920 metres.



Total Length = 1920 metres

(a) Chris, an ice-cream salesman, travels along each road at least once, starting and finishing at the point A. Find the length of an optimal ‘Chinese postman’ route for Chris. *(6 marks)*

(b) Pascal, a paperboy, starts at A and walks along each road at least once before finishing at D. Find the length of an optimal route for Pascal. *(2 marks)*

(c) Millie is to walk along all the roads at least once delivering leaflets. She can start her journey at any point and she can finish her journey at any point.

(i) Find the length of an optimal route for Millie. *(2 marks)*

(ii) State the points from which Millie could start in order to achieve this optimal route. *(1 mark)*

Turn over ▶

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

A factory makes two types of lock, standard and large, on a particular day.

On that day:

the maximum number of standard locks that the factory can make is 100;
the maximum number of large locks that the factory can make is 80;
the factory must make at least 60 locks in total;
the factory must make more large locks than standard locks.

Each standard lock requires 2 screws and each large lock requires 8 screws, and on that day the factory must use at least 320 screws.

On that day, the factory makes x standard locks and y large locks.

Each standard lock costs £1.50 to make and each large lock costs £3 to make.

The manager of the factory wishes to minimise the cost of making the locks.

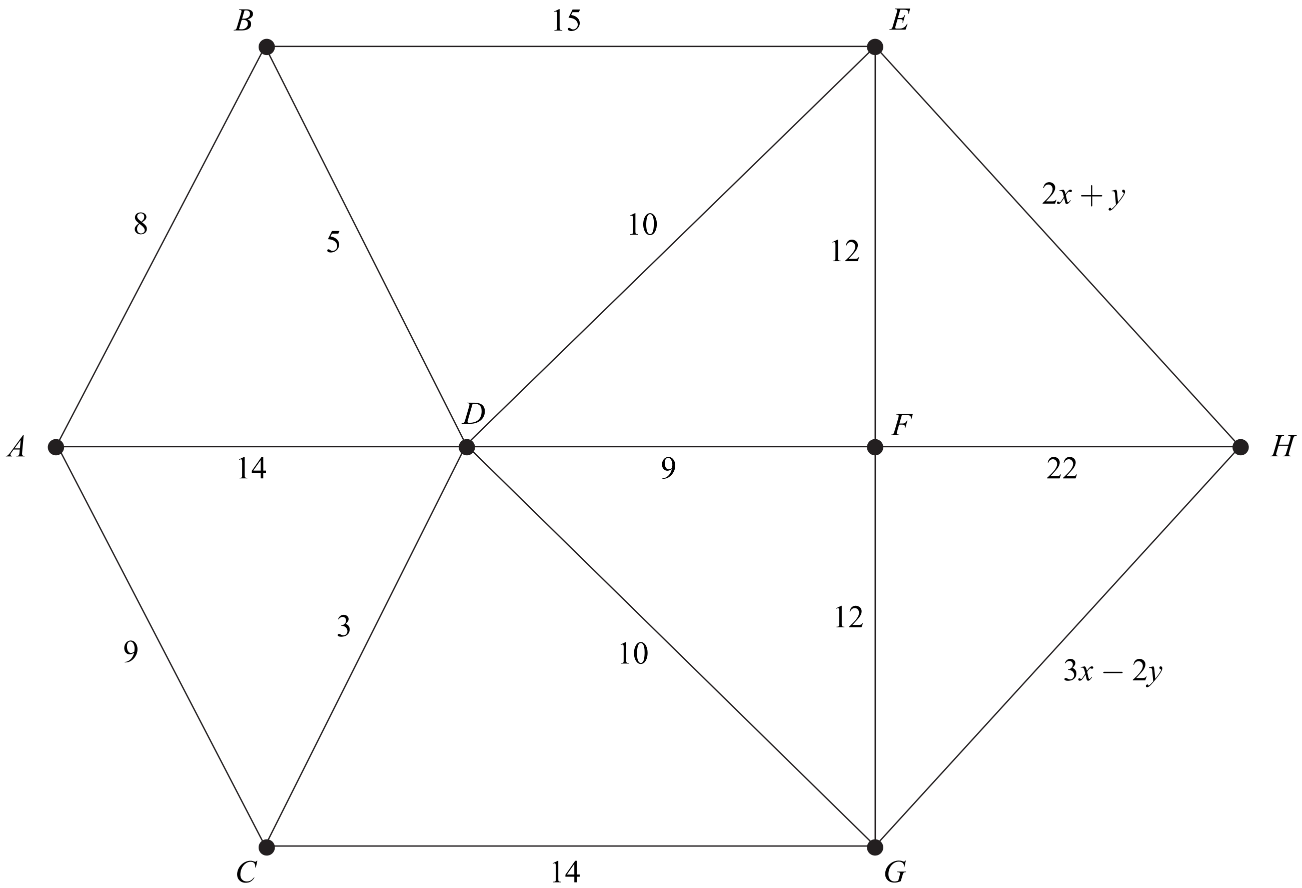
(a) Formulate the manager's situation as a linear programming problem. *(5 marks)*

(b) On **Figure 1**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*

(c) Find the values of x and y that correspond to the minimum cost. Hence find this minimum cost. *(4 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The following network has eight vertices, A, B, ..., H, and edges connecting some pairs of vertices. The number on each edge is its weight. The weights on the edges EH and GH are functions of x and y.



Given that there are three routes from A to H with the same minimum weight, use Dijkstra's algorithm on **Figure 2** to find:

(a) this minimum weight; *(6 marks)*

(b) the values of x and y. *(3 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Insert

Insert for use in **Questions 6 and 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

P5653/Jun08/MD01 6/6/6/

Figure 1 (for use in Question 6)

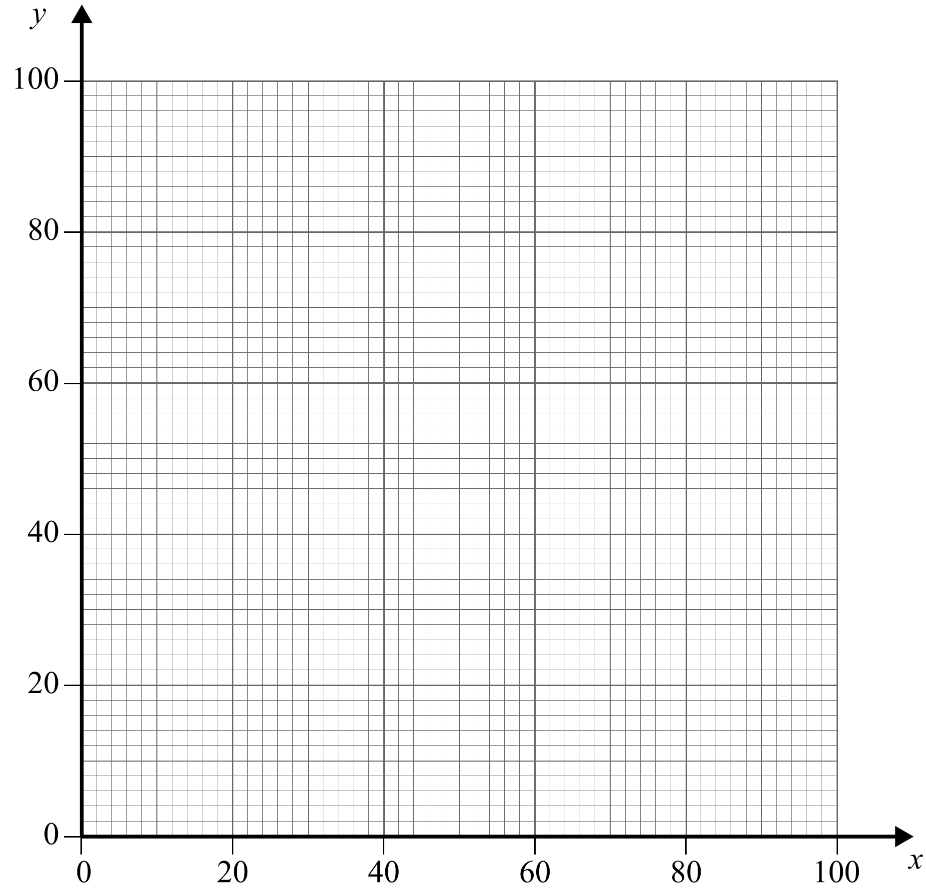
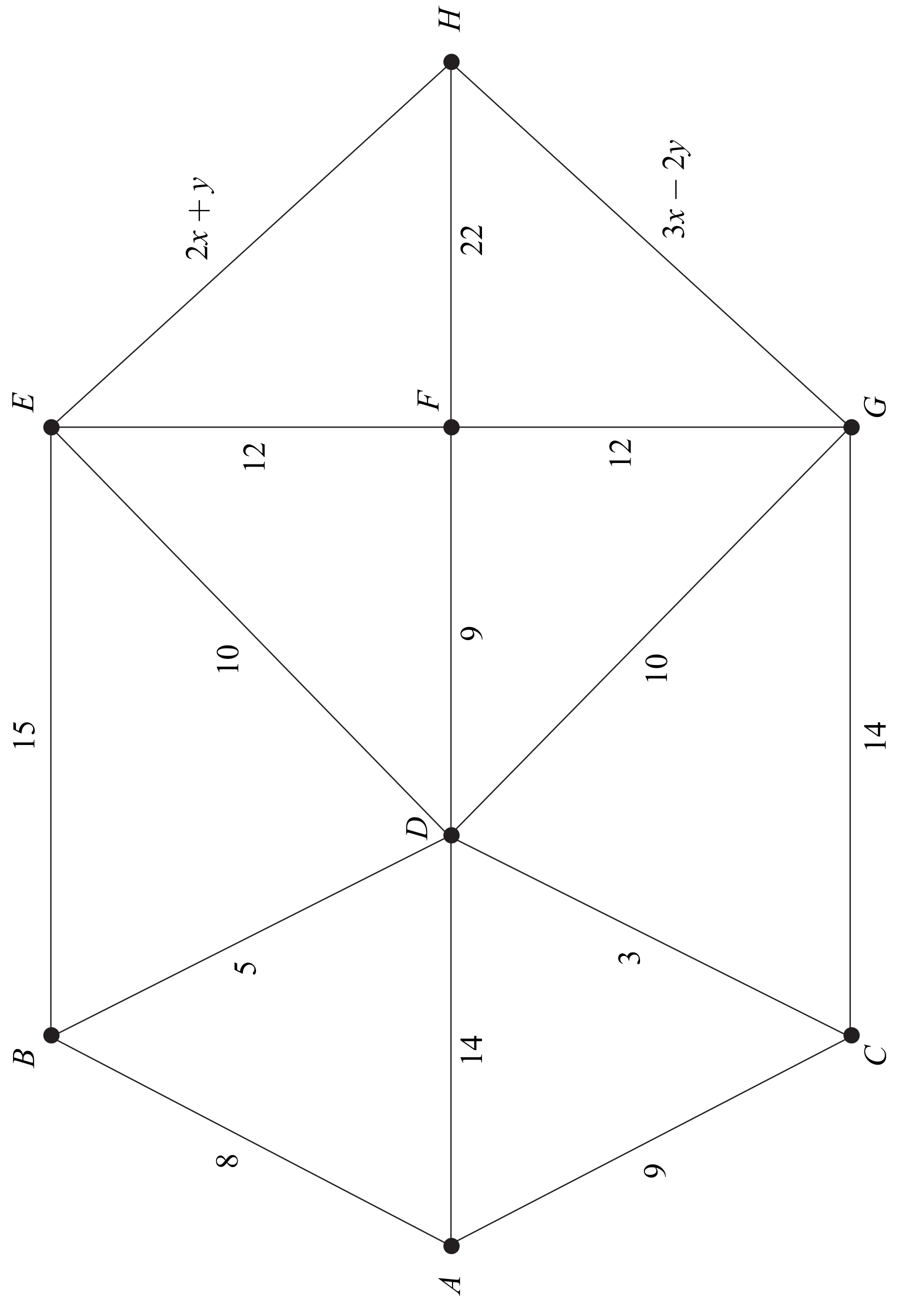


Figure 2 (for use in Question 7)



General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Wednesday 21 January 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 3 and 4 (enclosed).

You may use a graphics calculator.

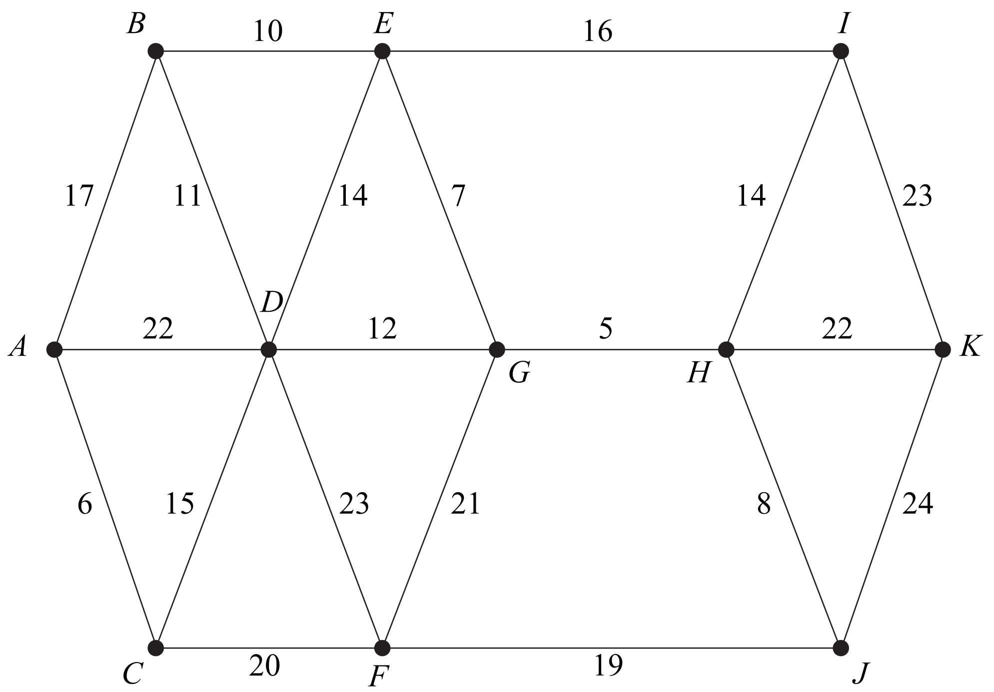
Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 The following network shows the lengths, in miles, of roads connecting 11 villages, A, B, ..., K.

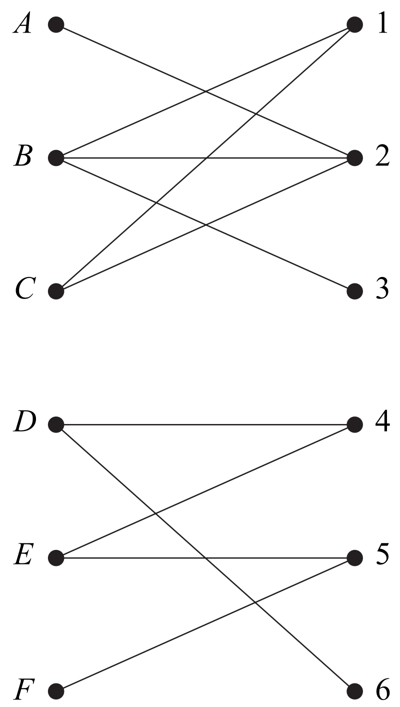


(a) Starting from G and showing your working at each stage, use Prim's algorithm to find a minimum spanning tree for the network. *(6 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

2 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

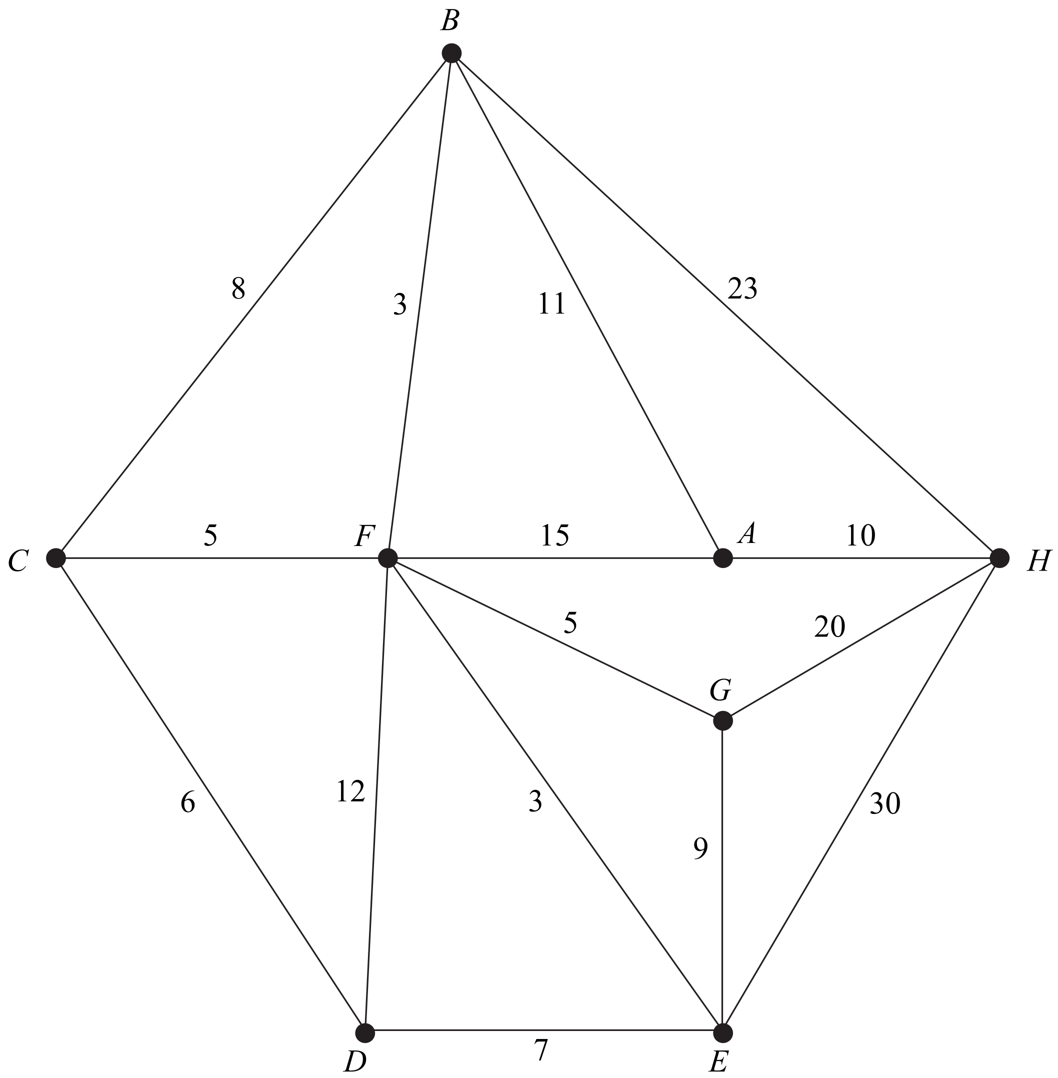
(b) Initially, B is assigned to task 1, C to task 2, D to task 4, and E to task 5.

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated to a task. *(5 marks)*

Turn over for the next question

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows roads connecting some places of interest in Berlin. The numbers represent the times taken, in minutes, to walk along the roads.



The total of all walking times is 167 minutes.

(a) Mia is staying at D and is to visit H.

(i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from D to H. *(6 marks)*

(ii) Write down the corresponding route. *(1 mark)*

(b) Each day, Leon has to deliver leaflets along all of the roads. He must start and finish at A.

(i) Use your answer to part (a) to write down the shortest walking time from D to A. *(1 mark)*

(ii) Find the walking time of an optimum Chinese Postman route for Leon. *(6 marks)*

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

Each year, farmer Giles buys some goats, pigs and sheep.

He must buy at least 110 animals.
He must buy at least as many pigs as goats.
The total of the number of pigs and the number of sheep that he buys must not be greater than 150.
Each goat costs £16, each pig costs £8 and each sheep costs £24.
He has £3120 to spend on the animals.

At the end of the year, Giles sells all of the animals. He makes a profit of £70 on each goat, £30 on each pig and £50 on each sheep. Giles wishes to maximize his total profit, £P.

Each year, Giles buys x goats, y pigs and z sheep.

(a) Formulate Giles's situation as a linear programming problem. *(5 marks)*

(b) One year, Giles buys 30 sheep.

(i) Show that the constraints for Giles's situation for this year can be modelled by

$$y \geqslant x, \qquad 2x + y \leqslant 300, \qquad x + y \geqslant 80, \qquad y \leqslant 120$$ *(2 marks)*

(ii) On **Figure 2**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(8 marks)*

(iii) Find Giles's maximum profit for this year and the number of each animal that he must buy to obtain this maximum profit. *(3 marks)*

Turn over for the next question

5 A student is using the algorithm below to find an approximate value of $\sqrt{2}$.

Line 10	Let $A = 1$, $B = 3$, $C = 0$
Line 20	Let $D = 1$, $E = 2$, $F = 0$
Line 30	Let $G = B/E$
Line 40	Let $H = G^2$
Line 50	If $(H - 2)^2 < 0.0001$ then go to Line 130
Line 60	Let $C = 2B + A$
Line 70	Let $A = B$
Line 80	Let $B = C$
Line 90	Let $F = 2E + D$
Line 100	Let $D = E$
Line 110	Let $E = F$
Line 120	Go to Line 30
Line 130	Print '$\sqrt{2}$ is approximately', B/E
Line 140	Stop

Trace the algorithm. *(6 marks)*

6 A connected graph G has five vertices and has eight edges with lengths 8, 10, 10, 11, 13, 17, 17 and 18.

(a) Find the minimum length of a minimum spanning tree for G. *(2 marks)*

(b) Find the maximum length of a minimum spanning tree for G. *(2 marks)*

(c) Draw a sketch to show a possible graph G when the length of the minimum spanning tree is 53. *(3 marks)*

7 Liam is taking part in a treasure hunt. There are five clues to be solved and they are at the points A, B, C, D and E. The table shows the distances between pairs of points. All of the distances are functions of x, **where x is an integer**.

Liam must travel to all five points, starting and finishing at A.

	A	**B**	**C**	**D**	**E**
A	–	$x + 6$	$2x - 4$	$3x - 7$	$4x - 14$
B	$x + 6$	–	$3x - 7$	$3x - 9$	$x + 9$
C	$2x - 4$	$3x - 7$	–	$2x - 1$	$x + 8$
D	$3x - 7$	$3x - 9$	$2x - 1$	–	$2x - 2$
E	$4x - 14$	$x + 9$	$x + 8$	$2x - 2$	–

(a) The nearest point to A is C.

(i) By considering AC and AB, show that $x < 10$. *(2 marks)*

(ii) Find two other inequalities in x. *(2 marks)*

(b) The nearest neighbour algorithm, starting from A, gives a **unique** minimum tour $ACDEBA$.

(i) By considering the fact that Liam's tour visits D immediately after C, find two further inequalities in x. *(3 marks)*

(ii) Find the value of the integer x. *(4 marks)*

(iii) Hence find the total distance travelled by Liam if he uses this tour. *(2 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2009

Mathematics MD01

Unit Decision 1

Specimen paper for examinations in June 2010 onwards

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the space provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 (a) Draw a bipartite graph representing the following adjacency matrix.

	1	2	3	4	5	6
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(2 marks)

(b) Initially, A is matched to 3, B is matched to 4, C is matched to 2, and E is matched to 5. Use the maximum matching algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

QUESTION PART REFERENCE

2 A student is using a shuttle sort to rearrange a set of numbers into ascending order.

Her correct solution is as follows.

Initial list	5	6	3	9	4	13	1
After 1st pass	5	6	3	9	4	13	1
After 2nd pass	3	5	6	9	4	13	1
After 3rd pass	3	5	6	9	4	13	1
After 4th pass	3	4	5	6	9	13	1
After 5th pass	3	4	5	6	9	13	1
After 6th pass	1	3	4	5	6	9	13

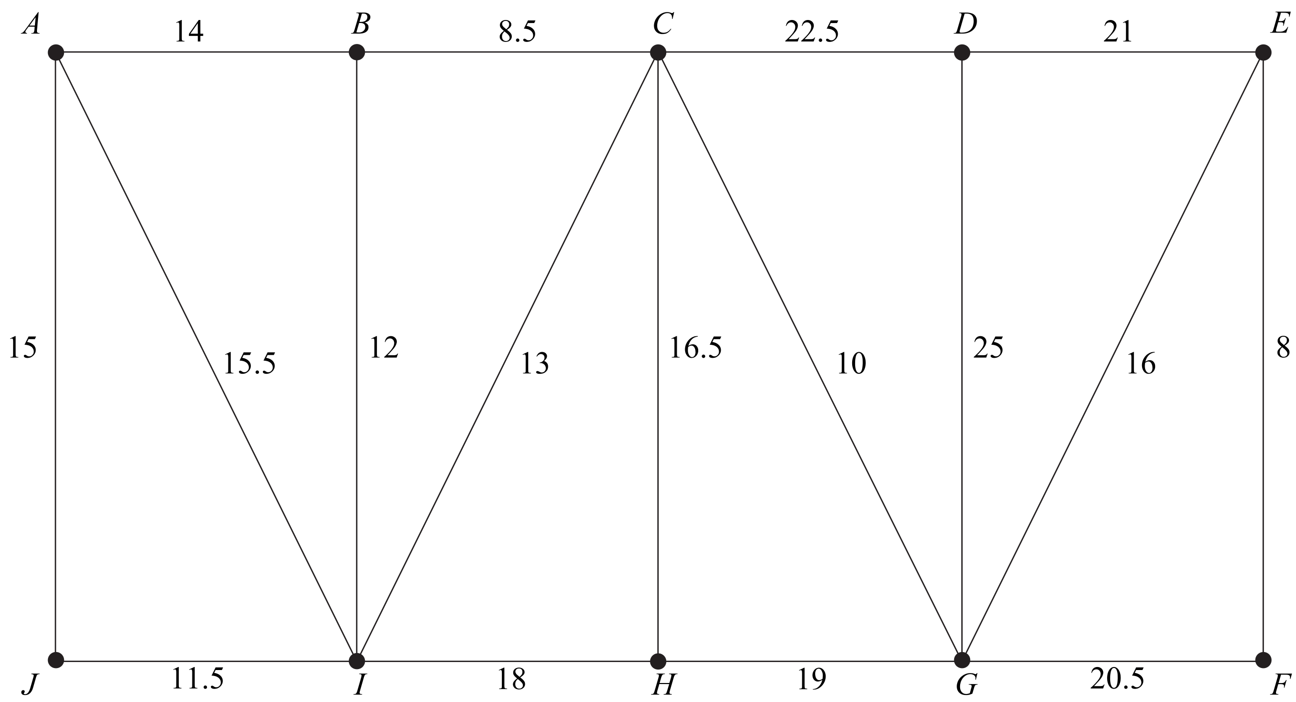
Write down the number of comparisons and swaps on each of the passes. *(6 marks)*

QUESTION PART REFERENCE

3 (a) (i) State the number of edges in a minimum spanning tree for a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, ..., J. The number on each edge represents the distance between a pair of adjacent vertices.



(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

4 The diagram opposite shows a network of roads on a housing estate. The number on each edge is the length, in metres, of the road.

Joe is starting a kitchen-fitting business.

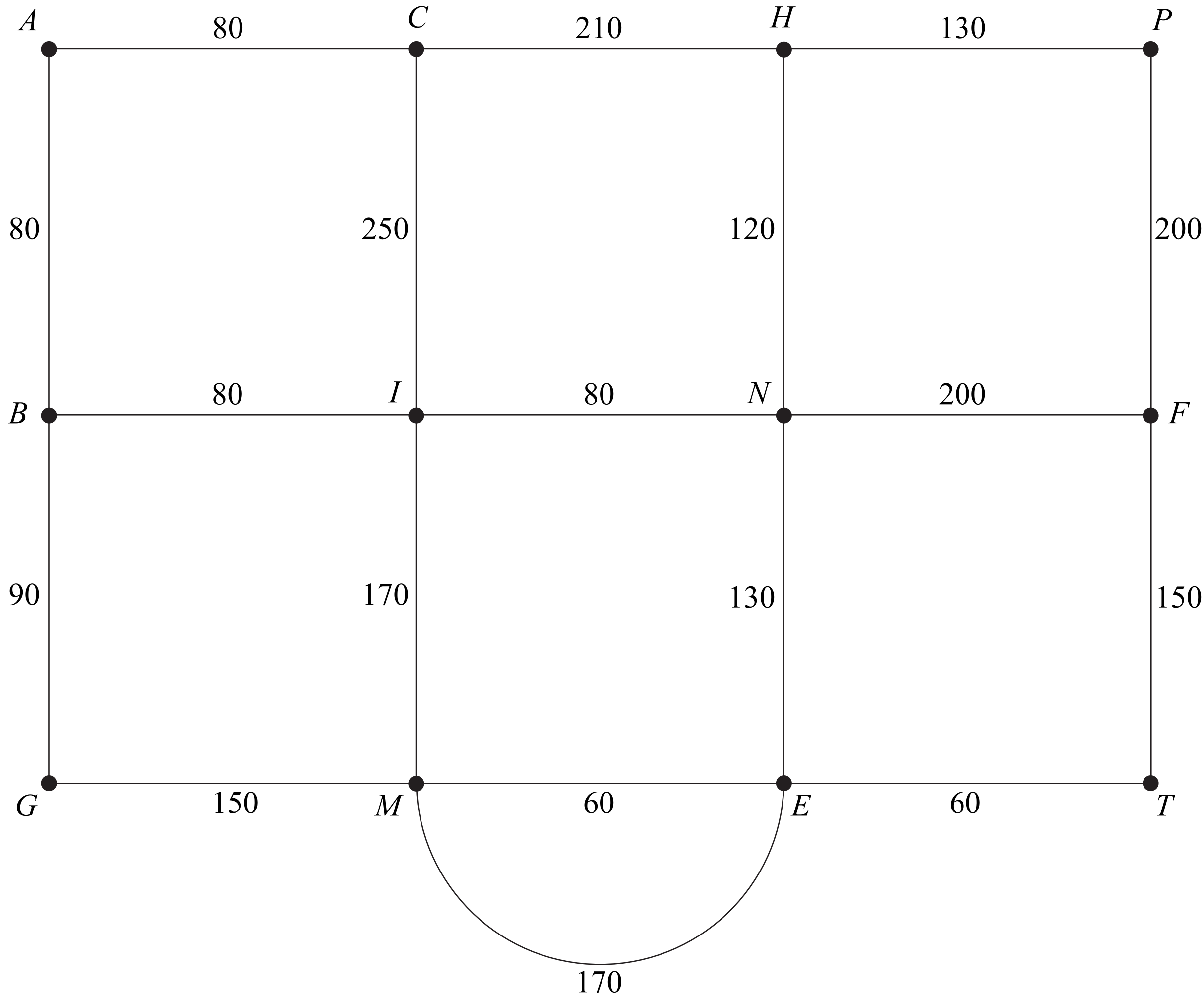
(a) Joe delivers leaflets advertising his business. He walks along all of the roads at least once, starting and finishing at C. Find the length of an optimal Chinese postman route for Joe. *(6 marks)*

(b) Joe gets a job fitting a kitchen in a house at T. Joe starts from C and wishes to drive to T. Use Dijkstra's algorithm on the diagram opposite to find the minimum distance to drive from C to T. State the corresponding route. *(7 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

(b)



Total length of roads = 2410 metres

5 Angelo is visiting six famous places in Palermo: A, B, C, D, E and F. He intends to travel from one place to the next until he has visited all of the places before returning to his starting place. Due to the traffic system, the time taken to travel between two places may be different dependent on the direction travelled.

The table shows the times, in minutes, taken to travel between the six places.

From \ To	***A***	***B***	***C***	***D***	***E***	***F***
A	–	25	20	20	27	25
B	15	–	10	11	15	30
C	5	30	–	15	20	19
D	20	25	15	–	25	10
E	10	20	7	15	–	15
F	25	35	29	20	30	–

(a) Give an example of a Hamiltonian cycle in this context. *(2 marks)*

(b) (i) Show that, if the nearest neighbour algorithm starting from F is used, the total travelling time for Angelo would be 95 minutes. *(3 marks)*

(ii) Explain why your answer to part **(b)(i)** is an upper bound for the minimum travelling time for Angelo. *(2 marks)*

(c) Angelo starts from F and visits E next. He also visits B before he visits D. Find an improved upper bound for Angelo's total travelling time. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

6 Each day, a factory makes three types of widget: basic, standard and luxury. The widgets produced need three different components: type A, type B and type C.

Basic widgets need 6 components of type A, 6 components of type B and 12 components of type C.
Standard widgets need 4 components of type A, 3 components of type B and 18 components of type C.
Luxury widgets need 2 components of type A, 9 components of type B and 6 components of type C.
Each day, there are 240 components of type A available, 300 of type B and 900 of type C.
Each day, the factory must use at least twice as many components of type C as type B.

Each day, the factory makes x basic widgets, y standard widgets and z luxury widgets.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find four inequalities in x, y and z that model the above constraints, simplifying each inequality. *(8 marks)*

(b) Each day, the factory makes the maximum possible number of widgets. On a particular day, the factory must make the same number of luxury widgets as basic widgets.

(i) Show that your answers in part **(a)** become

$$2x + y \leqslant 60, \quad 5x + y \leqslant 100, \quad x + y \leqslant 50, \quad y \geqslant x$$ *(3 marks)*

(ii) Using the axes opposite, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region. *(5 marks)*

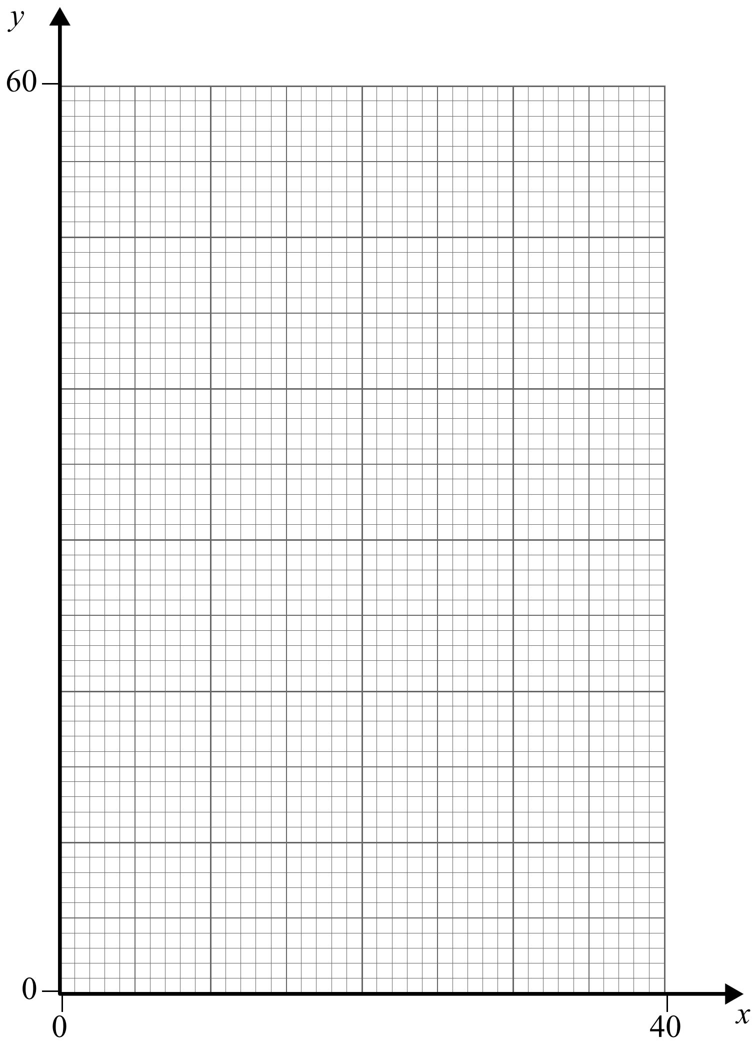
(iii) Find the total number of widgets made on that day. *(2 marks)*

(iv) Find all possible combinations of the number of each type of widget made that correspond to this maximum number. *(3 marks)*

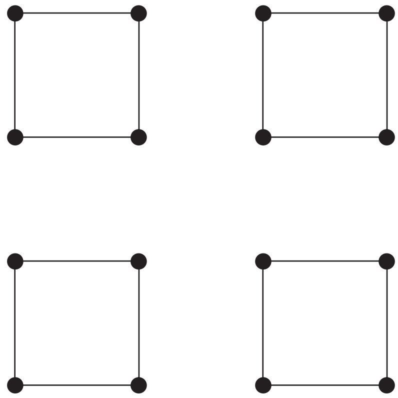
QUESTION PART REFERENCE

QUESTION PART REFERENCE

(b)



7 (a) The diagram shows a graph with 16 vertices and 16 edges.



(i) On **Figure 1** below, add the minimum number of edges to make a connected graph. *(1 mark)*

(ii) On **Figure 2** opposite, add the minimum number of edges to make the graph Hamiltonian. *(2 marks)*

(iii) On **Figure 3** opposite, add the minimum number of edges to make the graph Eulerian. *(2 marks)*

(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

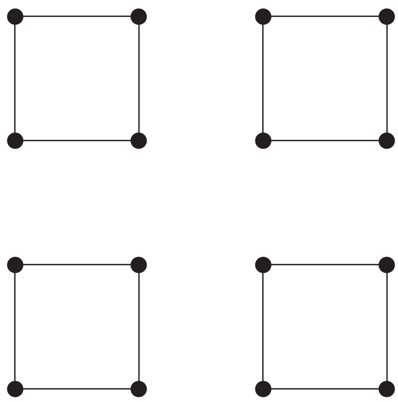
(ii) The number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(2 marks)*

QUESTION PART REFERENCE

(a)(i)

Figure 1

Connected Graph

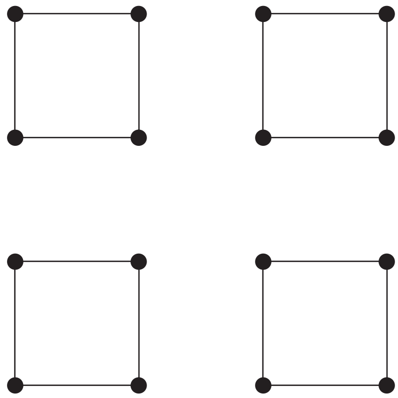


QUESTION PART REFERENCE

(a)(ii)

Figure 2

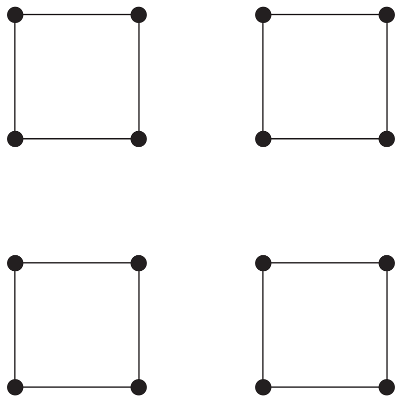
Hamiltonian Graph



(a)(iii)

Figure 3

Eulerian Graph



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END OF QUESTIONS

Surname		Other Names										
Centre Number						Candidate Number						
Candidate Signature												

General Certificate of Education
June 2009
Advanced Subsidiary Examination

MATHEMATICS **MD01**
Unit Decision 1

Insert

Insert for use in **Questions 4, 6 and 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 4)

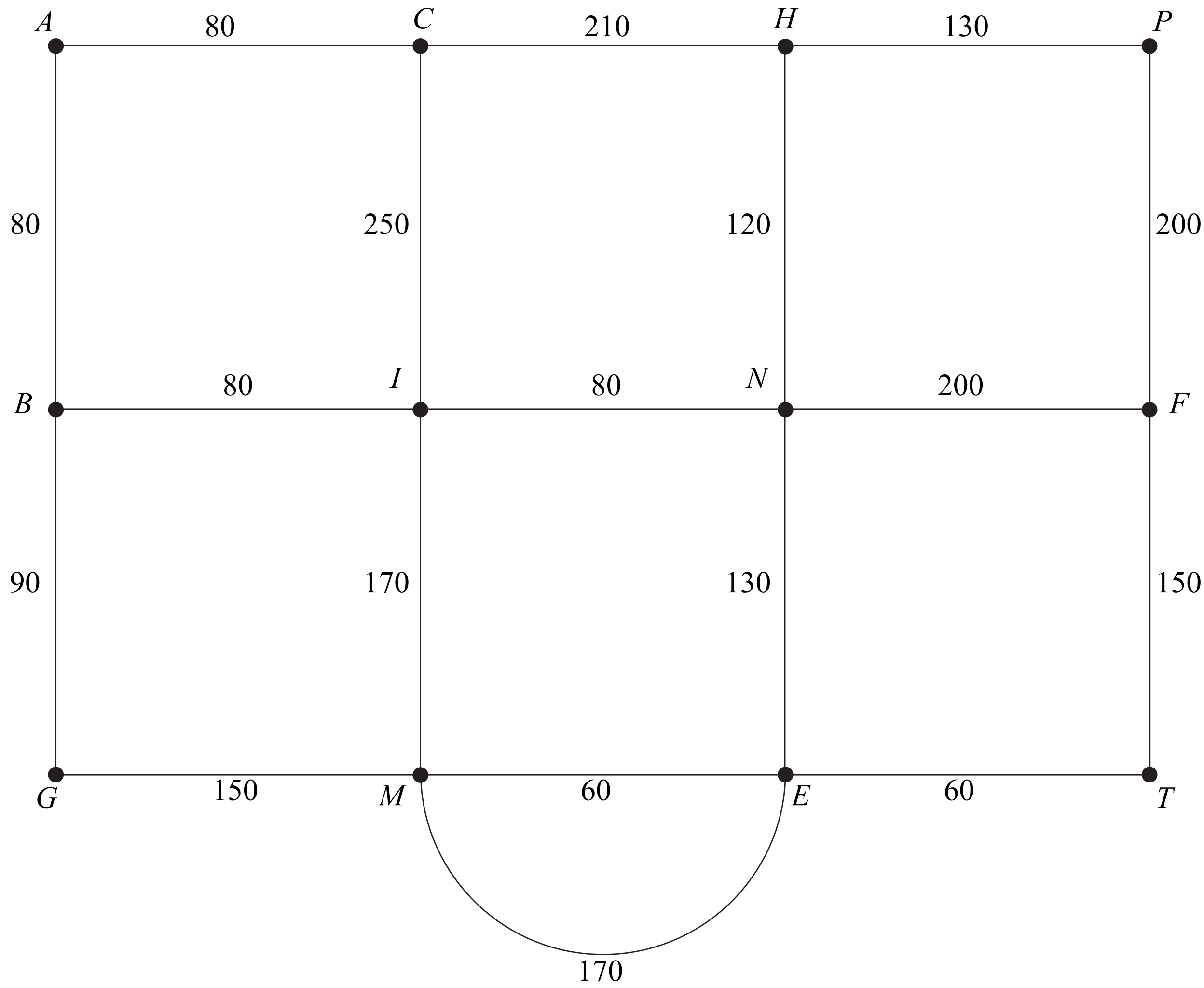


Figure 2 (for use in Question 6)

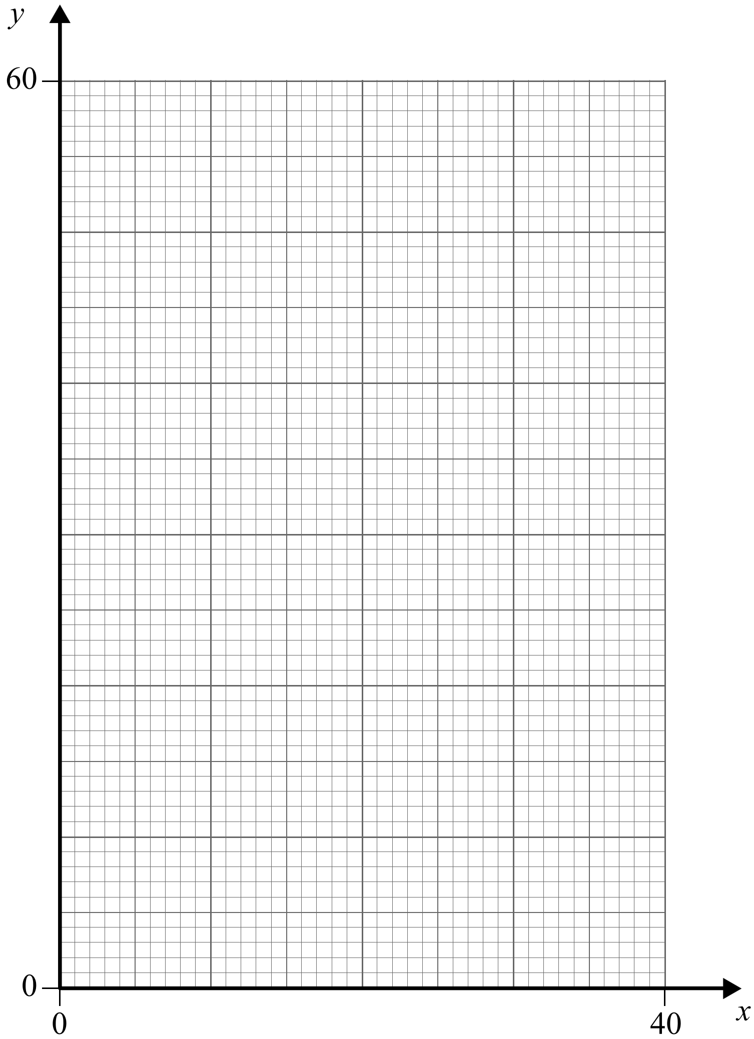


Figure 3 (for use in Question 7(a)(i))

Connected Graph

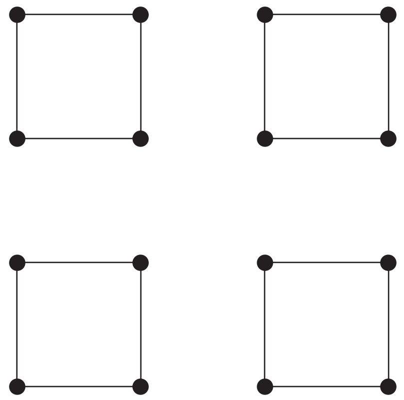


Figure 4 (for use in Question 7(a)(ii))

Hamiltonian Graph

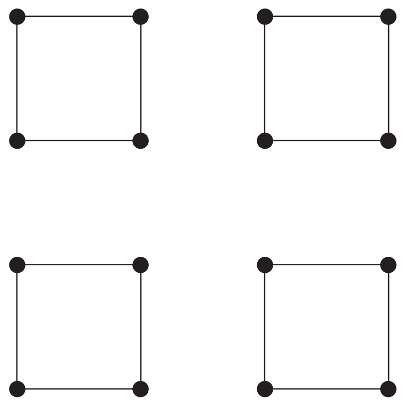
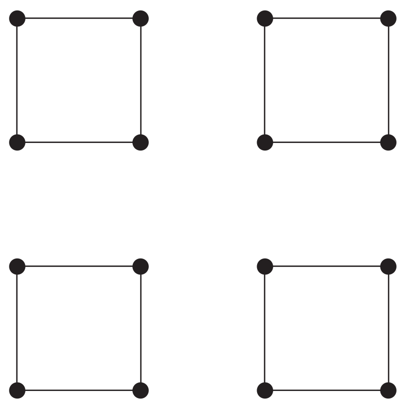


Figure 5 (for use in Question 7(a)(iii))

Eulerian Graph



General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MD01

Unit Decision 1

Tuesday 19 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 3 and 7 (enclosed).

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MD01.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions.

1 Six girls, Alfonsa (A), Bianca (B), Claudia (C), Desiree (D), Erika (E) and Flavia (F), are going to a pizza restaurant. The restaurant provides a special menu of six different pizzas: Margherita (M), Neapolitana (N), Pepperoni (P), Romana (R), Stagioni (S) and Viennese (V).

The table shows the pizzas that each girl likes.

Girl	**Pizza**
Alfonsa (A)	Margherita (M), Pepperoni (P), Stagioni (S)
Bianca (B)	Neapolitana (N), Romana (R)
Claudia (C)	Neapolitana (N), Viennese (V)
Desiree (D)	Romana (R), Stagioni (S)
Erika (E)	Pepperoni (P), Stagioni (S), Viennese (V)
Flavia (F)	Romana (R)

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Each girl is to eat a different pizza. Initially, the waiter brings six different pizzas and gives Alfonsa the Pepperoni, Bianca the Romana, Claudia the Neapolitana and Erika the Stagioni. The other two pizzas are put in the middle of the table.

From this initial matching, use the maximum matching algorithm to obtain a complete matching so that every girl gets a pizza that she likes. List your complete matching. *(5 marks)*

2 (a) Use a bubble sort to rearrange the following numbers into ascending order.

13 16 10 11 4 12 6 7 *(5 marks)*

(b) State the number of comparisons and the number of swaps (exchanges) for each of the first three passes. *(3 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by the following:

$$x \geqslant 0,\ y \geqslant 0$$
$$x + 4y \leqslant 36$$
$$4x + y \leqslant 68$$
$$y \leqslant 2x$$
$$y \geqslant \frac{1}{4}x$$

(a) On **Figure 1**, draw a suitable diagram to represent the inequalities and indicate the feasible region. *(6 marks)*

(b) Use your diagram to find the maximum value of P, stating the corresponding coordinates, on the feasible region, in the case where:

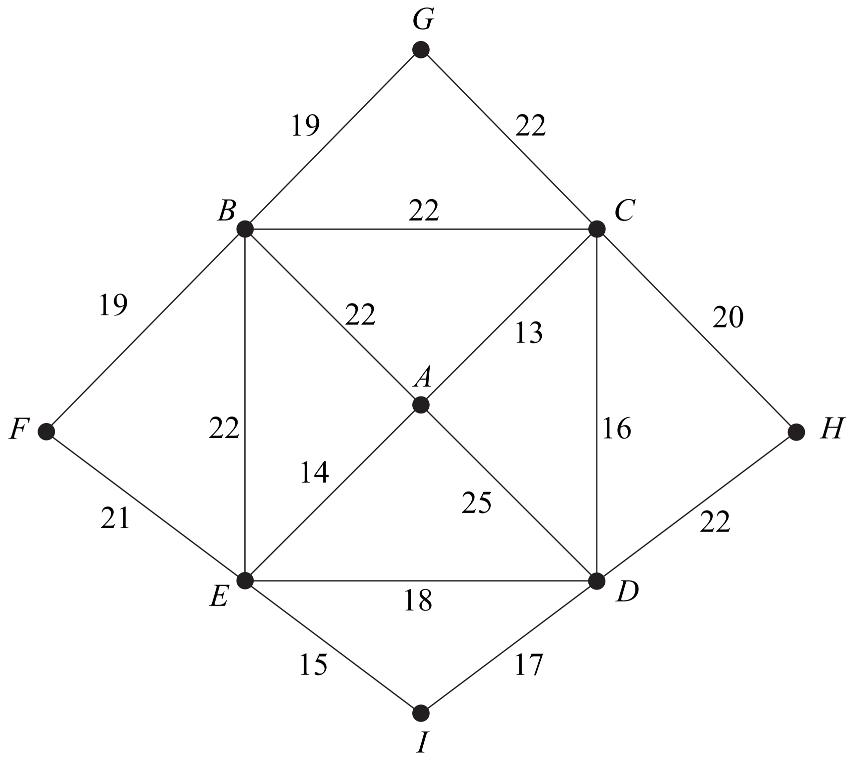
(i) $P = x + 5y$; *(2 marks)*

(ii) $P = 5x + y$. *(2 marks)*

Turn over for the next question

4 In Paris, there is a park where there are statues of famous people; there are many visitors each day to this park. Lighting is to be installed at nine places, A, B, …, I, in the park. The places have to be connected either directly or indirectly by cabling, to be laid alongside the paths, as shown in the diagram.

The diagram shows the length of each path, in metres, connecting adjacent places.



Total length of paths = 307 metres

(a) (i) Use Prim's algorithm, starting from A, to find the minimum length of cabling required. *(5 marks)*

(ii) State this minimum length. *(1 mark)*

(iii) Draw the minimum spanning tree. *(2 marks)*

(b) A security guard walks along all the paths before returning to his starting place. Find the length of an optimal Chinese postman route for the guard. *(6 marks)*

5 There is a one-way system in Manchester. Mia is parked at her base, B, in Manchester and intends to visit four other places, A, C, D and E, before returning to her base. The following table shows the distances, in kilometres, for Mia to drive between the five places A, B, C, D and E. Mia wants to keep the total distance that she drives to a minimum.

From \ To	*A*	*B*	*C*	*D*	*E*
A	–	1.7	1.9	1.8	2.1
B	3.1	–	2.5	1.8	3.7
C	3.1	2.9	–	2.7	4.2
D	2.0	2.8	2.1	–	2.3
E	2.2	3.6	1.9	1.7	–

(a) Find the length of the tour *BECDAB*. *(1 mark)*

(b) Find the length of the tour obtained by using the nearest neighbour algorithm starting from B. *(4 marks)*

(c) Write down which of your answers to parts (a) and (b) would be the better upper bound for the total distance that Mia drives. *(1 mark)*

(d) On a particular day, the council decides to reverse the one-way system. For this day, find the length of the tour obtained by using the nearest neighbour algorithm starting from B. *(4 marks)*

Turn over for the next question

6 A student is finding a numerical approximation for the area under a curve.

The algorithm that the student is using is as follows:

Line 10	Input A, B, N
Line 20	Let $T = 0$
Line 30	Let $D = A$
Line 40	Let $H = (B - A)/N$
Line 50	Let $E = H/2$
Line 60	Let $T = T + A^3 + B^3$
Line 70	Let $D = D + H$
Line 80	If $D = B$ then go to line 110
Line 90	Let $T = T + 2D^3$
Line 100	Go to line 70
Line 110	Print 'Area = ', $T \times E$
Line 120	End

Trace the algorithm in the case where the input values are:

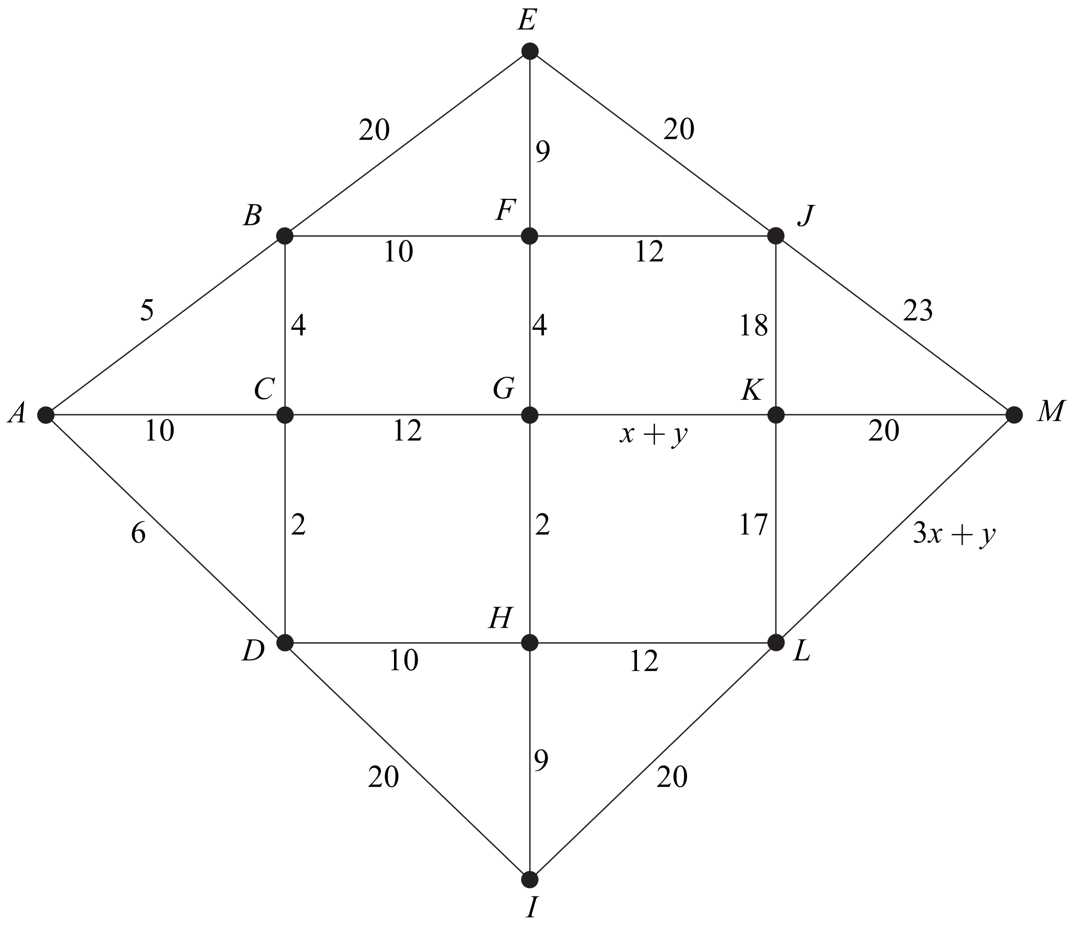
(a) $A = 1$, $B = 5$, $N = 2$; *(4 marks)*

(b) $A = 1$, $B = 5$, $N = 4$. *(4 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The following network has 13 vertices and 24 edges connecting some pairs of vertices. The number on each edge is its weight.

The weights on the edges GK and LM are functions of x and y, where $x > 0$, $y > 0$ and $10 < x + y < 27$.



There are three routes from A to M of the same minimum total weight.

(a) Use Dijkstra's algorithm on **Figure 2** to find this minimum total weight. *(7 marks)*

(b) Find the values of x and y. *(3 marks)*

Turn over for the next question

8 A factory packs three different kinds of novelty box: red, blue and green. Each box contains three different types of toy: A, B and C.

Each red box has 2 type A toys, 3 type B toys and 4 type C toys.

Each blue box has 3 type A toys, 1 type B toy and 3 type C toys.

Each green box has 4 type A toys, 5 type B toys and 2 type C toys.

Each day, the maximum number of each type of toy available to be packed is 360 type A, 300 type B and 400 type C.

Each day, the factory must pack more type A toys than type B toys.

Each day, the total number of type A and type B toys that are packed must together be at least as many as the number of type C toys that are packed.

Each day, at least 40% of the total toys that are packed must be type C toys.

Each day, the factory packs x red boxes, y blue boxes and z green boxes.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, simplifying your answers. *(8 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MD01

Unit Decision 1

Wednesday 9 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 (a) Draw a bipartite graph representing the following adjacency matrix.

	1	**2**	**3**	**4**	**5**
A	1	0	0	0	1
B	0	1	1	1	0
C	0	1	1	1	0
D	1	0	0	0	1
E	1	0	0	0	1

(2 marks)

(b) If A, B, C, D and E represent five people and 1, 2, 3, 4 and 5 represent five tasks to which they are to be assigned, explain why a complete matching is impossible.

(2 marks)

QUESTION PART REFERENCE

Do not write outside the box

2 (a) (i) Use a bubble sort to rearrange the following numbers into ascending order, showing the list of numbers after each pass.

6 2 3 5 4 *(3 marks)*

(ii) Write down the number of comparisons on the first pass. *(1 mark)*

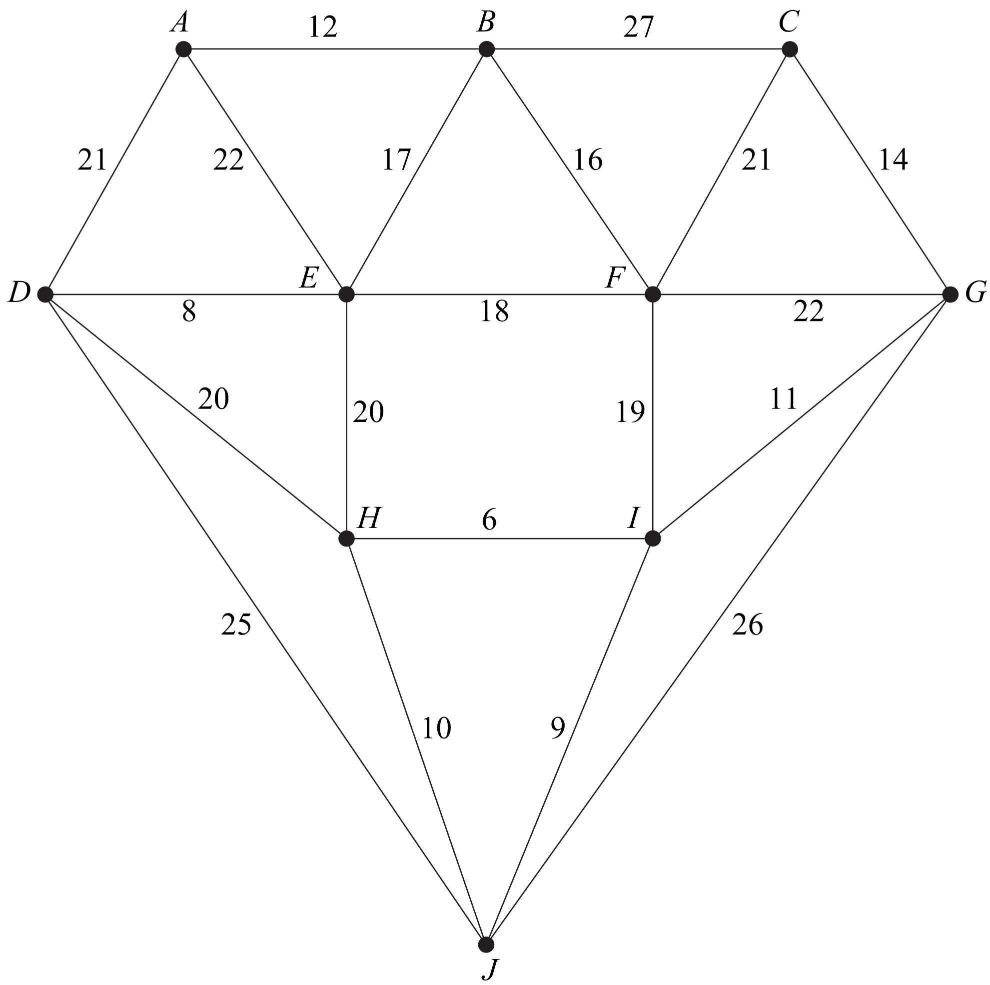
(b) (i) Use a shuttle sort to rearrange the following numbers into ascending order, showing the list of numbers after each pass.

6 2 3 5 4 *(4 marks)*

(ii) Write down the number of comparisons on the first pass. *(1 mark)*

QUESTION PART REFERENCE

3 The network shows 10 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 towns. *(6 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(d) If Prim's algorithm, starting at B, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(1 mark)*

QUESTION PART REFERENCE

4 The network below shows 13 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.

The total of all the times is 384 minutes.

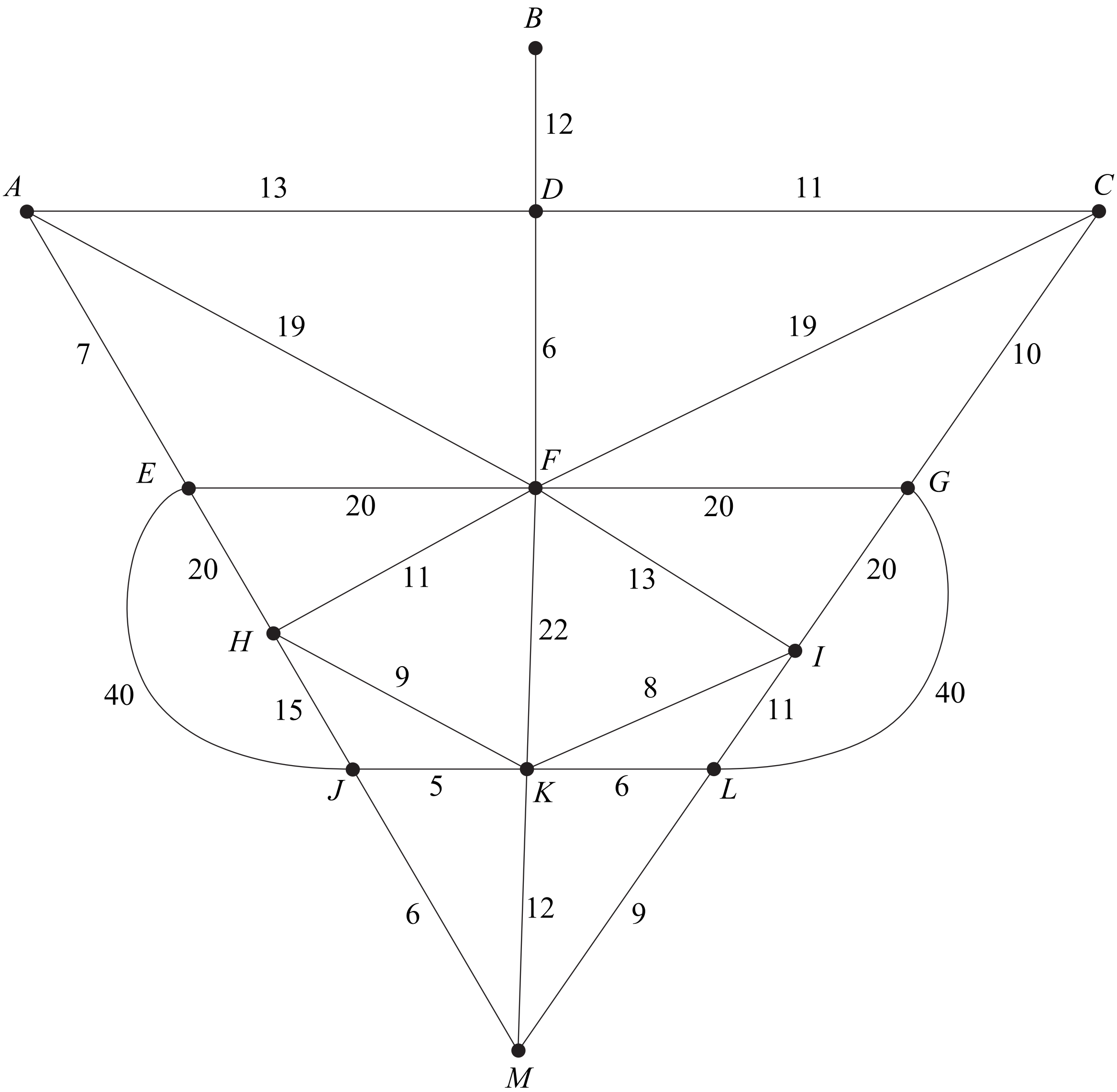
(a) Use Dijkstra's algorithm on the network below, starting from M, to find the minimum time to travel from M to each of the other towns. *(7 marks)*

(b) (i) Find the travelling time of an optimum Chinese postman route around the network, starting and finishing at M. *(6 marks)*

(ii) State the number of times that the vertex F would appear in a corresponding route. *(1 mark)*

QUESTION PART REFERENCE

(a)



5 Phil, a squash coach, wishes to buy some equipment for his club. In a town centre, there are six shops, G, I, N, R, S and T, that sell the equipment.

The time, in seconds, to walk between each pair of shops is shown in the table.

Phil intends to check prices by visiting each of the six shops before returning to his starting point.

	G	***I***	***N***	***R***	***S***	***T***
G	–	81	82	86	72	76
I	81	–	80	82	68	73
N	82	80	–	84	70	74
R	86	82	84	–	74	70
S	72	68	70	74	–	64
T	76	73	74	70	64	–

(a) Use the nearest neighbour algorithm starting from S to find an upper bound for Phil's minimum walking time. *(4 marks)*

(b) Write down a tour starting from N which has a total walking time equal to your answer to part **(a)**. *(1 mark)*

(c) By deleting S, find a lower bound for Phil's minimum walking time. *(5 marks)*

QUESTION PART REFERENCE

6 Phil is to buy some squash balls for his club. There are three different types of ball that he can buy: slow, medium and fast.

He must buy at least 190 slow balls, at least 50 medium balls and at least 50 fast balls.

He must buy at least 300 balls in total.

Each slow ball costs £2.50, each medium ball costs £2.00 and each fast ball costs £2.00.

He must spend no more than £1000 in total.

At least 60% of the balls that he buys must be slow balls.

Phil buys x slow balls, y medium balls and z fast balls.

(a) Find six inequalities that model Phil's situation. *(4 marks)*

(b) Phil decides to buy the same number of medium balls as fast balls.

(i) Show that the inequalities found in part **(a)** simplify to give

$$x \geqslant 190, \quad y \geqslant 50, \quad x + 2y \geqslant 300, \quad 5x + 8y \leqslant 2000, \quad y \leqslant \frac{1}{3}x$$ *(2 marks)*

(ii) Phil sells all the balls that he buys to members of the club. He sells each slow ball for £3.00, each medium ball for £2.25 and each fast ball for £2.25. He wishes to maximise his profit.

On **Figure 1** on page 14, draw a diagram to enable this problem to be solved graphically, indicating the feasible region and the direction of an objective line. *(7 marks)*

(iii) Find Phil's maximum possible profit and state the number of each type of ball that he must buy to obtain this maximum profit. *(4 marks)*

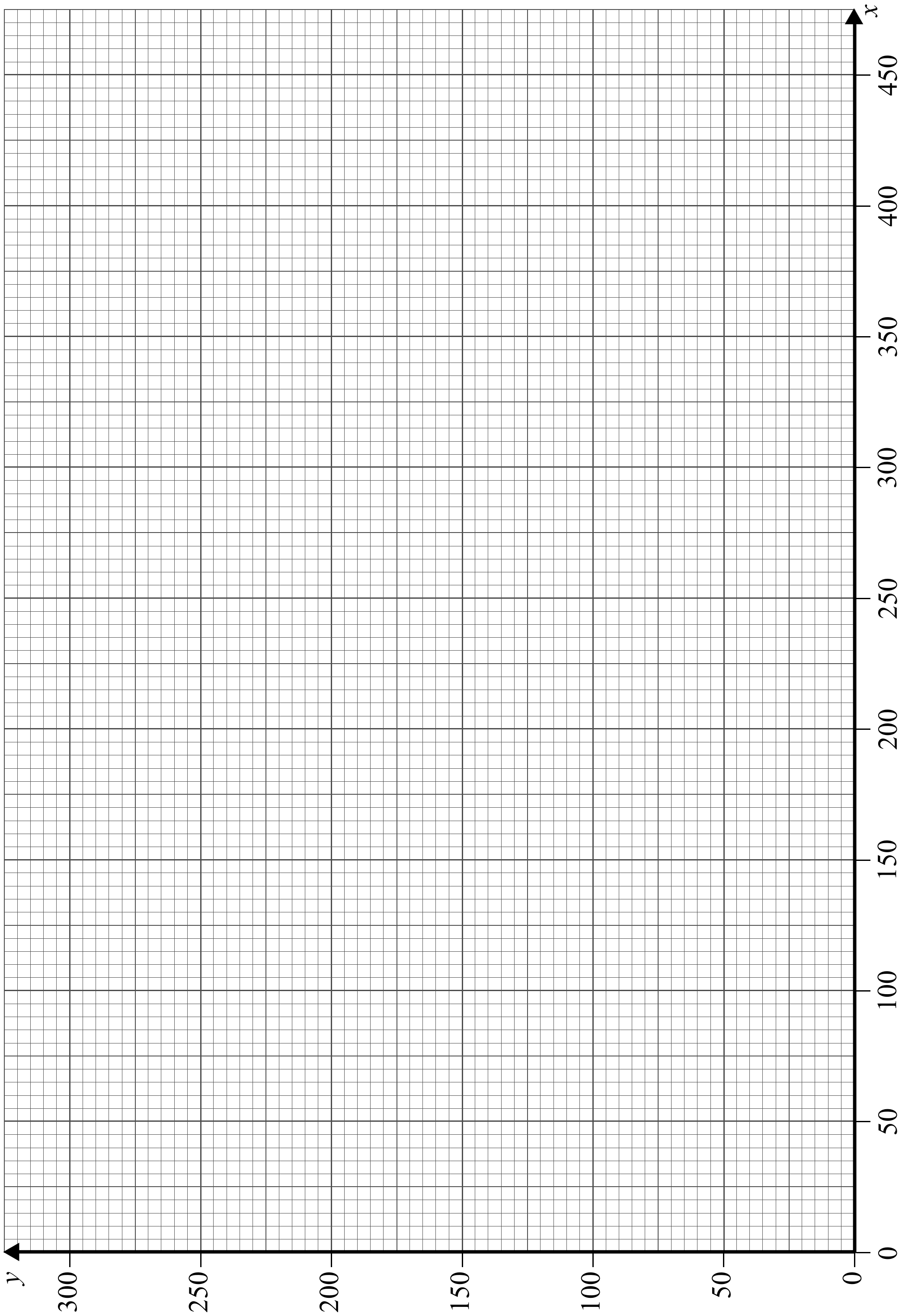
QUESTION PART REFERENCE

QUESTION PART REFERENCE

(b)(ii)

Figure 1

$x \geqslant 190, \;\; y \geqslant 50, \;\; x + 2y \geqslant 300, \;\; 5x + 8y \leqslant 2000, \;\; y \leqslant \frac{1}{3}x$



7 A student is testing a numerical method for finding an approximation for π.

The algorithm that the student is using is as follows.

Line 10	Input A, B, C, D, E
Line 20	Let $A = A + 2$
Line 30	Let $B = -B$
Line 40	Let $C = \dfrac{B}{A}$
Line 50	Let $D = D + C$
Line 60	Let $E = (D - 3.14)^2$
Line 70	If $E < 0.05$ then go to Line 90
Line 80	Go to Line 20
Line 90	Print 'π is approximately', D
Line 100	End

Trace the algorithm in the case where the input values are

$$A = 1,\ B = 4,\ C = 0,\ D = 4,\ E = 0$$

(6 marks)

QUESTION PART REFERENCE

8 A simple connected graph has six vertices.

(a) One vertex has degree x. State the greatest and least possible values of x. *(2 marks)*

(b) The six vertices have degrees

$$x-2,\ x-2,\ x,\ 2x-4,\ 2x-4,\ 4x-12$$

Find the value of x, justifying your answer. *(2 marks)*

QUESTION PART REFERENCE

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
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TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MD01

Unit Decision 1

Monday 24 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

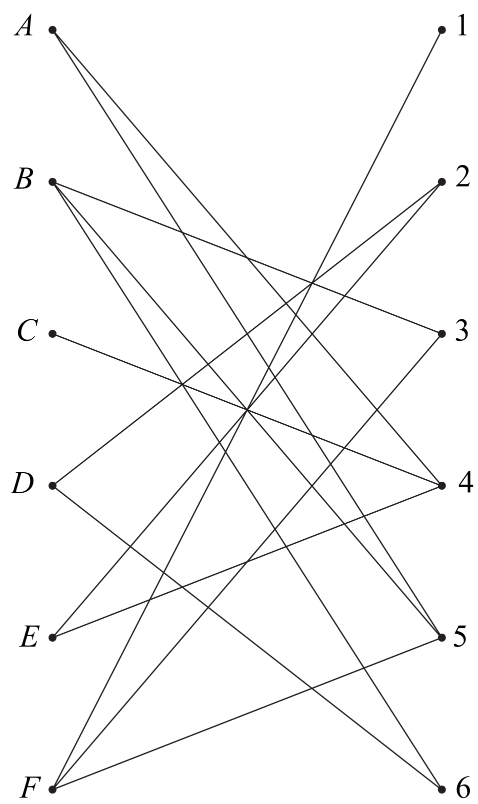
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

(b) Initially, B is assigned to task 5, D to task 2, E to task 4 and F to task 3.

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated to a task. *(5 marks)*

QUESTION PART REFERENCE

2 A student is using a quicksort algorithm to rearrange a set of numbers into ascending order. She uses the first number in each list (or sublist) as the pivot.

Her correct solution for the first three passes is as follows.

Initial list	10	7	4	22	13	16	19	5
After 1st pass	7	4	5	10	22	13	16	19
After 2nd pass	4	5	7	10	13	16	19	22
After 3rd pass	4	5	7	10	13	16	19	22

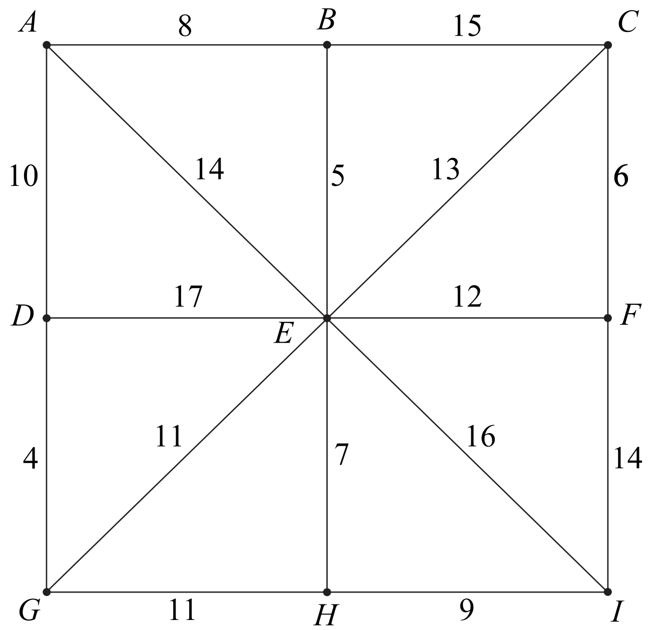
(a) State the pivots used for the 2nd pass. *(2 marks)*

(b) Write down the number of comparisons on each of the three passes. *(3 marks)*

(c) Explain whether the student has completed the algorithm. *(1 mark)*

QUESTION PART REFERENCE

3 The following network shows the lengths, in miles, of roads connecting nine villages, A, B, …, I.



(a) (i) Use Prim's algorithm starting from E, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

(b) On a particular day, village B is cut off, so its connecting roads cannot be used.

Find the length of a minimum spanning tree for the remaining eight villages. *(2 marks)*

QUESTION PART REFERENCE

4 The network below shows some paths on an estate. The number on each edge represents the time taken, in minutes, to walk along a path.

(a) (i) Use Dijkstra's algorithm on the network to find the minimum walking time from A to J. *(6 marks)*

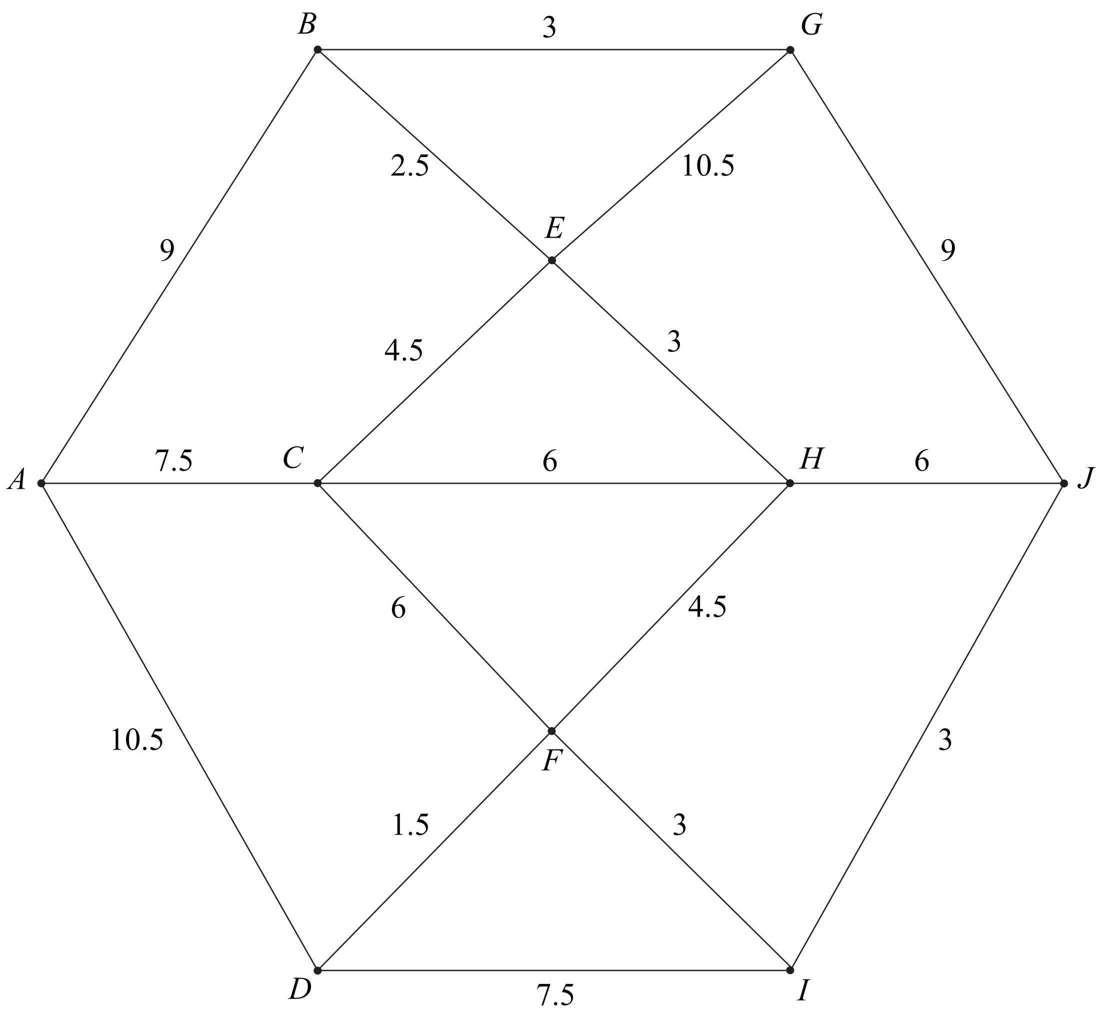
(ii) Write down the corresponding route. *(1 mark)*

(b) A new subway is constructed connecting C to G directly. The time taken to walk along this subway is x minutes. The minimum time taken to walk from A to G is now reduced, but the minimum time taken to walk from A to J is not reduced.

Find the range of possible values for x. *(3 marks)*

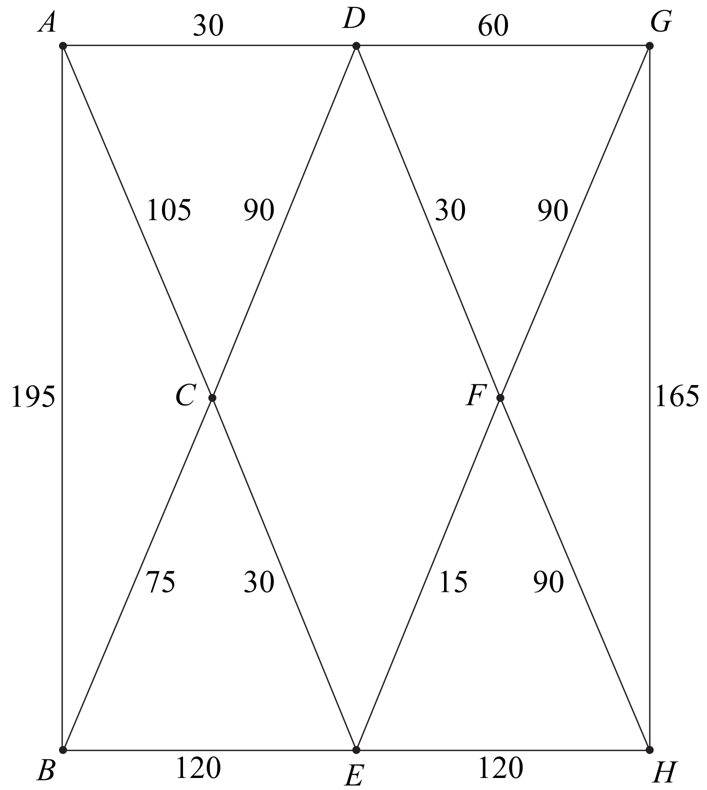
QUESTION PART REFERENCE

(a)(i)



5 Norris delivers newspapers to houses on an estate. The network shows the streets on the estate. The number on each edge shows the length of the street, in metres.

Norris starts from the newsagents located at vertex *A*, and he must walk along all the streets at least once before returning to the newsagents.



The total length of the streets is 1215 metres.

(a) Give a reason why it is not possible to start at *A*, walk along each street once only, and return to *A*. *(1 mark)*

(b) Find the length of an optimal Chinese postman route around the estate, starting and finishing at *A*. *(5 marks)*

(c) For an optimal Chinese postman route, state:

(i) the number of times that the vertex *F* would occur; *(1 mark)*

(ii) the number of times that the vertex *H* would occur. *(1 mark)*

Do not write outside the box

6 (a) The complete graph K_n has every one of its n vertices connected to each of the other vertices by a single edge.

(i) Find the total number of edges in the graph K_5. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for the graph K_5. *(1 mark)*

(iii) State the number of edges in a Hamiltonian cycle for the graph K_5. *(1 mark)*

(b) A simple graph G has six vertices and nine edges, and G is Eulerian. Draw a sketch to show a possible graph G. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

7 Fred delivers bread to five shops, A, B, C, D and E. Fred starts his deliveries at shop B, and travels to each of the other shops once before returning to shop B. Fred wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	A	***B***	***C***	***D***	***E***
A	–	3	11	15	5
B	3	–	18	12	4
C	11	18	–	5	16
D	15	12	5	–	10
E	5	4	16	10	–

(a) Find the travelling time for the tour $BACDEB$. *(1 mark)*

(b) Use the nearest neighbour algorithm, starting at B, to find another upper bound for the travelling time for Fred's tour. *(3 marks)*

(c) By deleting C, find a lower bound for the travelling time for Fred's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

8 A student is tracing the following algorithm with positive integer values of A and B.

The function INT gives the integer part of a number, eg $\text{INT}(2.3) = 2$ and $\text{INT}(3.8) = 3$.

Line 10	Let $X = 0$
Line 20	Input A, B
Line 30	If $\text{INT}(A/2) = A/2$ then go to Line 50
Line 40	Let $X = X + B$
Line 50	If $A = 1$ then go to Line 90
Line 60	Let $A = \text{INT}(A/2)$
Line 70	Let $B = 2 \times B$
Line 80	Go to Line 30
Line 90	Print X
Line 100	End

(a) Trace the algorithm in the case where the input values are $A = 20$ and $B = 8$. *(4 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

(c) Another student changed Line 50 to

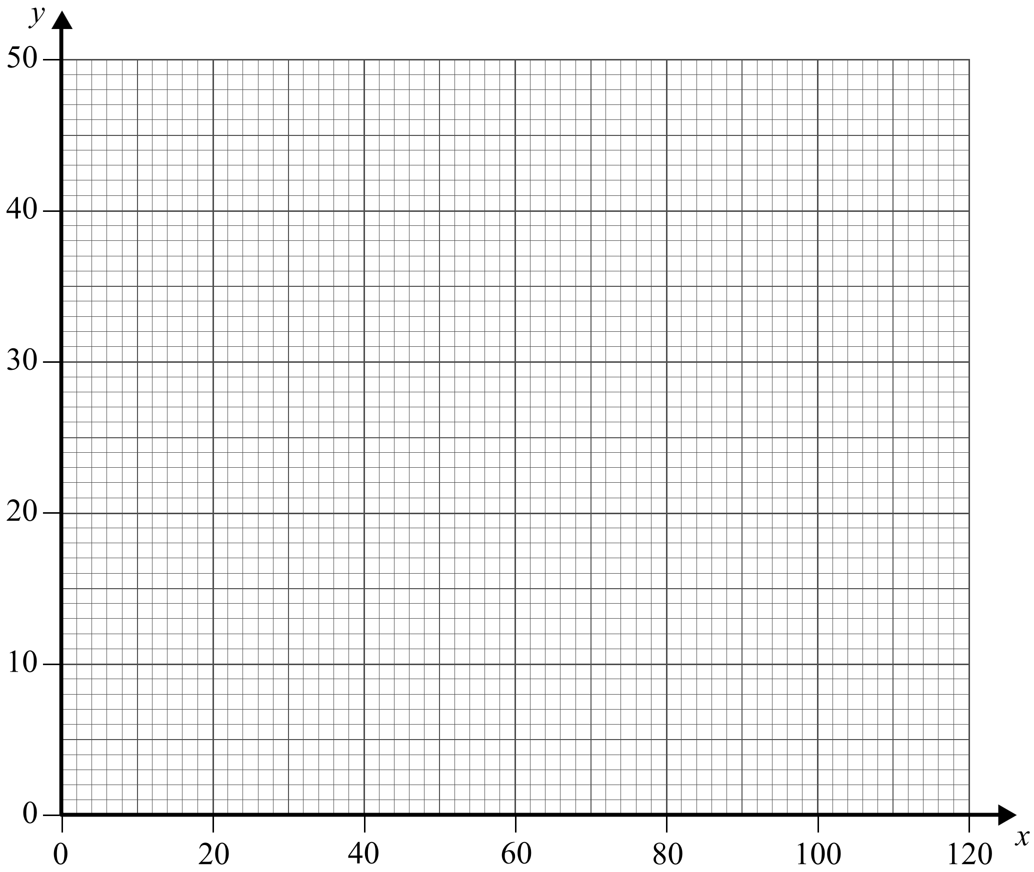
Line 50 If $A = 1$ then go to Line 80

Explain what would happen if this algorithm were traced. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

(b)(ii)



Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
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General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MD01

Unit Decision 1

Thursday 26 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the tasks that each of the people is able to undertake.

	1	**2**	**3**	**4**	**5**	**6**
A	1	0	1	0	0	0
B	1	1	0	0	0	1
C	0	1	0	1	0	0
D	0	1	0	0	1	0
E	0	0	1	0	1	0
F	0	0	0	0	1	0

(a) Represent this information in a bipartite graph. *(2 marks)*

(b) Initially, A is assigned to task 3, B to task 2, C to task 4 and D to task 5.

Use an algorithm from this initial matching to find a maximum matching, listing your alternating paths. *(5 marks)*

QUESTION PART REFERENCE

Do not write outside the box

2 Five different integers are to be sorted into ascending order.

(a) A bubble sort is to be used on the list of numbers $6 \quad 4 \quad x \quad 2 \quad 11$.

(i) After the first pass, the list of numbers becomes

$$4 \quad x \quad 2 \quad 6 \quad 11$$

Write down an inequality that x must satisfy. *(1 mark)*

(ii) After the second pass, the list becomes

$$x \quad 2 \quad 4 \quad 6 \quad 11$$

Write down a new inequality that x must satisfy. *(1 mark)*

(b) The five integers are now written in a different order. A shuttle sort is to be used on the list of numbers $11 \quad x \quad 2 \quad 4 \quad 6$.

(i) After the first pass, the list of numbers becomes

$$x \quad 11 \quad 2 \quad 4 \quad 6$$

Write down an inequality that x must satisfy. *(1 mark)*

(ii) After the second pass, the list becomes

$$2 \quad x \quad 11 \quad 4 \quad 6$$

Write down a further inequality that x must satisfy. *(1 mark)*

(c) Use your answers from parts **(a)** and **(b)** to write down the value of x. *(2 marks)*

QUESTION PART REFERENCE

3 A group of eight friends, A, B, C, D, E, F, G and H, keep in touch by sending text messages. The cost, in pence, of sending a message between each pair of friends is shown in the following table.

	A	***B***	***C***	***D***	***E***	***F***	***G***	***H***
A	–	15	10	12	16	11	14	17
B	15	–	15	14	15	16	16	15
C	10	15	–	11	10	12	14	9
D	12	14	11	–	11	12	14	12
E	16	15	10	11	–	13	15	14
F	11	16	12	12	13	–	14	8
G	14	16	14	14	15	14	–	13
H	17	15	9	12	14	8	13	–

One of the group wishes to pass on a piece of news to all the other friends, either by a direct text or by the message being passed on from friend to friend, at the minimum total cost.

(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the table. *(4 marks)*

(ii) Draw your minimum spanning tree. *(2 marks)*

(iii) Find the minimum total cost. *(1 mark)*

(b) Person H leaves the group. Find the new minimum total cost. *(2 marks)*

QUESTION PART REFERENCE

4 The network below shows some pathways at a school connecting different departments. The number on each edge represents the time taken, in minutes, to walk along that pathway.

Carol, the headteacher, wishes to walk from her office (O) to the Drama department (D).

(a) (i) Use Dijkstra's algorithm on the network to find the minimum walking time from O to D. *(6 marks)*

(ii) Write down the corresponding route. *(1 mark)*

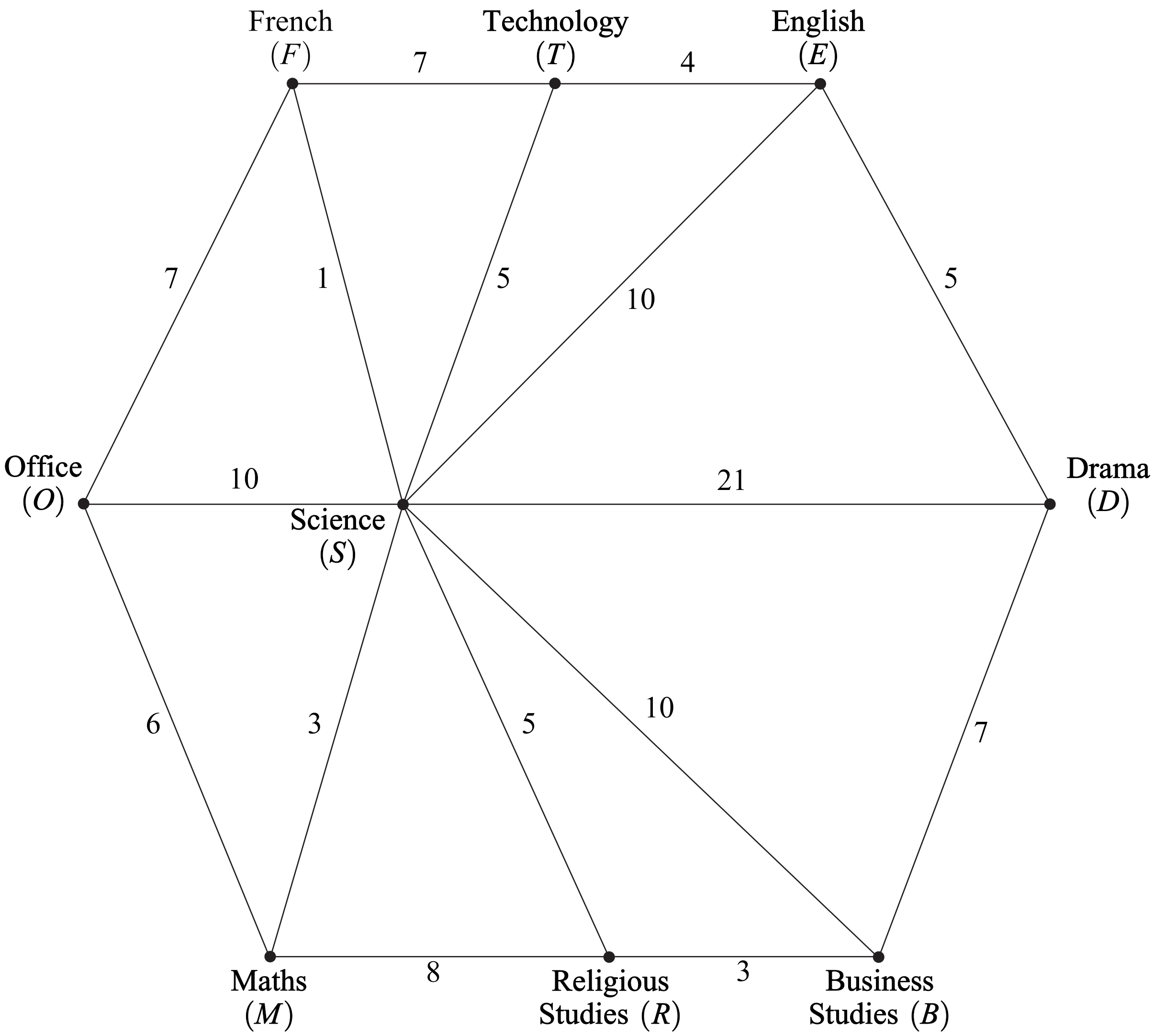
(b) On another occasion, Carol needs to go from her office to the Business Studies department (B).

(i) Write down her minimum walking time. *(1 mark)*

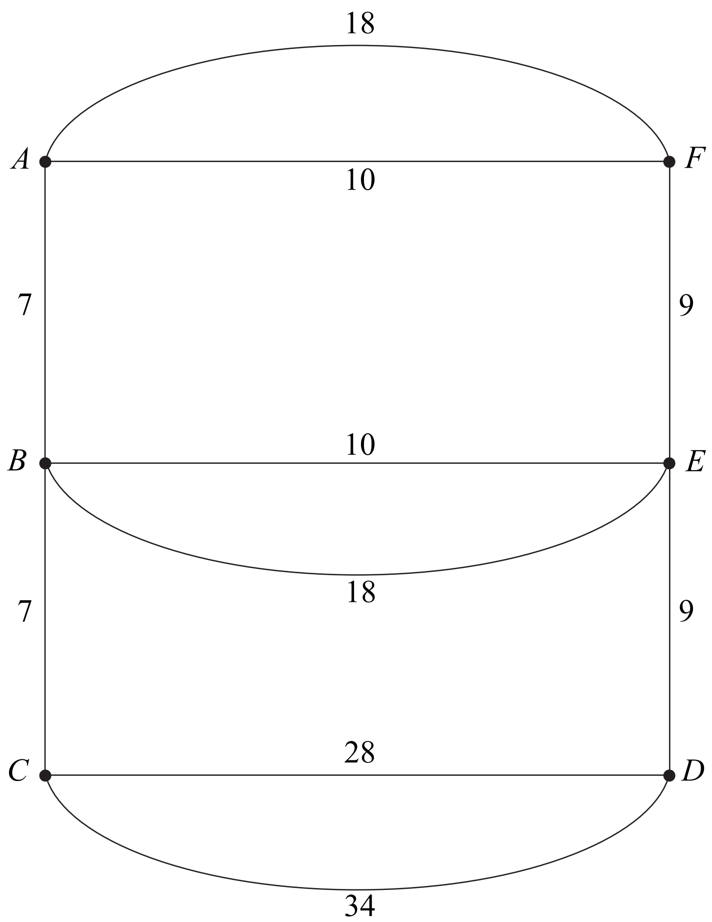
(ii) Write down the route corresponding to this minimum time. *(1 mark)*

QUESTION PART REFERENCE

(a)(i)



5 A council is responsible for gritting main roads in a district. The network shows the main roads in the district. The number on each edge shows the length of the road, in kilometres. The gritter starts from the depot located at point A, and must drive along all the roads at least once before returning to the depot.



Total $= 150$ km

(a) Find the length of an optimal Chinese postman route around the main roads in the district, starting and finishing at A. *(5 marks)*

(b) Zac, a supervisor, wishes to inspect all the roads. He leaves the depot, located at point A, and drives along all the roads at least once before finishing at his home, located at point C. Find the length of an optimal route for Zac. *(2 marks)*

(c) Liz, a reporter, intends to drive along all the roads at least once in order to report on driving conditions. She can start her journey at any point and can finish her journey at any point.

(i) Find the length of an optimal route for Liz. *(2 marks)*

(ii) State the points from which Liz could start in order to achieve this optimal route. *(1 mark)*

6 A student is tracing the following algorithm.

Line 10	Let $A = 6$
Line 20	Let $B = 7$
Line 30	Input C
Line 40	Let $D = (A + B)/2$
Line 50	Let $E = C - D^3$
Line 60	If $E^2 < 1$ then go to Line 120
Line 70	If $E > 0$ then go to Line 100
Line 80	Let $B = D$
Line 90	Go to Line 40
Line 100	Let $A = D$
Line 110	Go to Line 40
Line 120	Stop

(a) Trace the algorithm in the case where the input value is $C = 300$. *(4 marks)*

(b) The algorithm is intended to find the approximate cube root of any input number.

State two reasons why the algorithm is unsatisfactory in its present form. *(3 marks)*

QUESTION PART REFERENCE

7 A builder needs some screws, nails and plugs. At the local store, there are packs containing a mixture of the three items.

A DIY pack contains 200 nails, 200 screws and 100 plugs.

A trade pack contains 1000 nails, 1500 screws and 2500 plugs.

A DIY pack costs £2.50 and a trade pack costs £15.

The builder needs at least 5000 nails, 6000 screws and 4000 plugs.

The builder buys x DIY packs and y trade packs and wishes to keep his total cost to a minimum.

(a) Formulate the builder's situation as a linear programming problem. *(4 marks)*

(b) (i) On the grid opposite, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of an objective line. *(6 marks)*

(ii) Use your diagram to find the number of each type of pack that the builder should buy in order to minimise his cost. *(1 mark)*

(iii) Find the builder's minimum cost. *(1 mark)*

QUESTION PART REFERENCE

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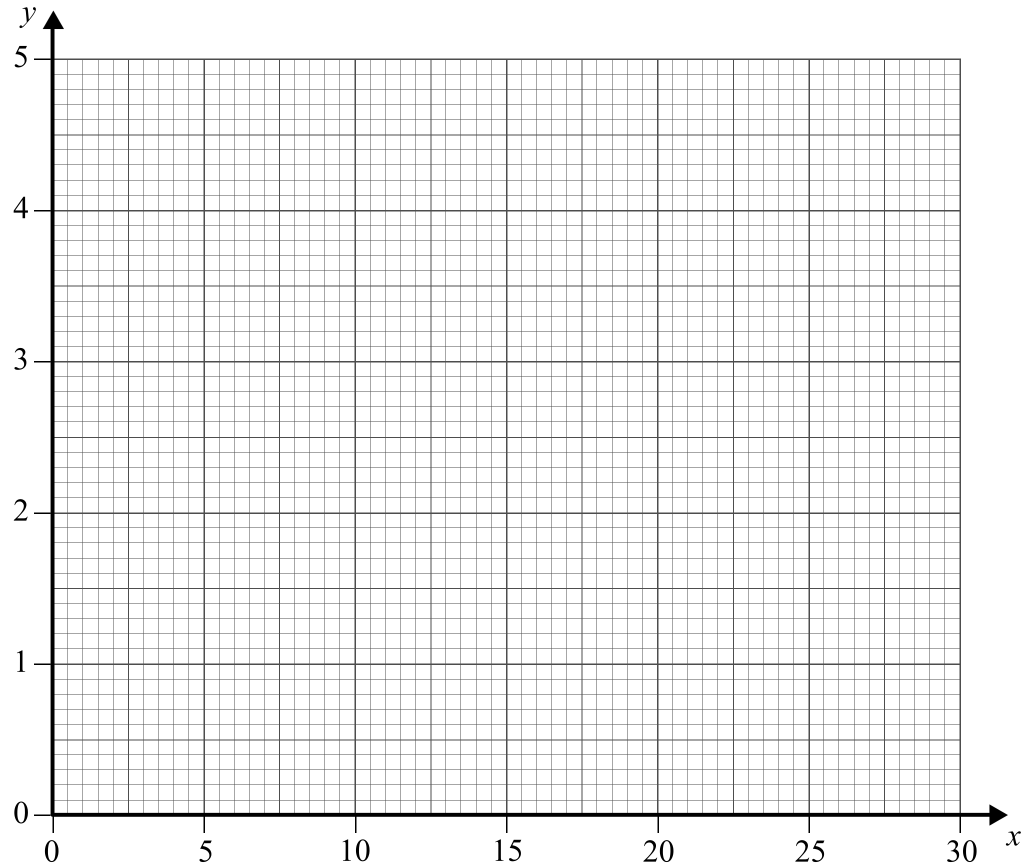
QUESTION PART REFERENCE

(b)(i)



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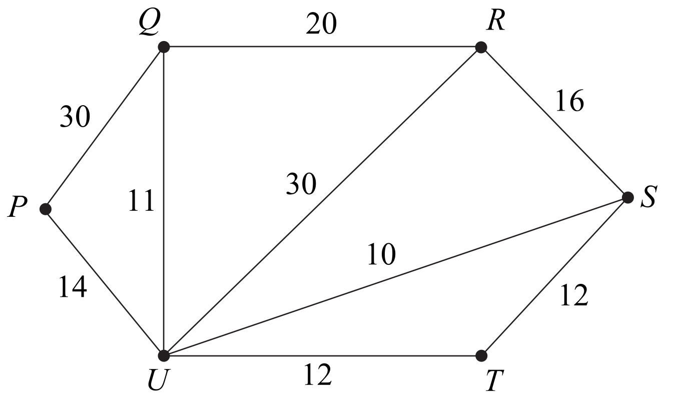
8 Mrs Jones is a spy who has to visit six locations, P, Q, R, S, T and U, to collect information. She starts at location Q, and travels to each of the other locations, before returning to Q. She wishes to keep her travelling time to a minimum.

The diagram represents roads connecting different locations. The number on each edge represents the travelling time, in minutes, along that road.



(a) (i) Explain why the shortest time to travel from P to R is 40 minutes. *(2 marks)*

(ii) Complete **Table 1**, on the opposite page, in which the entries are the shortest travelling times, in minutes, between pairs of locations. *(2 marks)*

(b) (i) Use the nearest neighbour algorithm on **Table 1**, starting at Q, to find an upper bound for the minimum travelling time for Mrs Jones's tour. *(4 marks)*

(ii) Mrs Jones decides to follow the route given by the nearest neighbour algorithm. Write down her route, showing all the locations through which she passes. *(2 marks)*

(c) By deleting Q from **Table 1**, find a lower bound for the travelling time for Mrs Jones's tour. *(5 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

(a)(ii)

Table 1

	P	***Q***	***R***	***S***	***T***	***U***
P	–	25				
Q	25	–	20	21	23	11
R		20	–			
S		21		–		
T		23			–	12
U		11			12	–

Turn over ▶

Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
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General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MD01

Unit Decision 1

Monday 23 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 Use a Shell sort to rearrange the following numbers into ascending order, showing the new arrangement after each pass.

37 25 16 12 36 24 13 11 *(5 marks)*

QUESTION PART REFERENCE

2 (a) Draw a bipartite graph representing the following adjacency matrix.

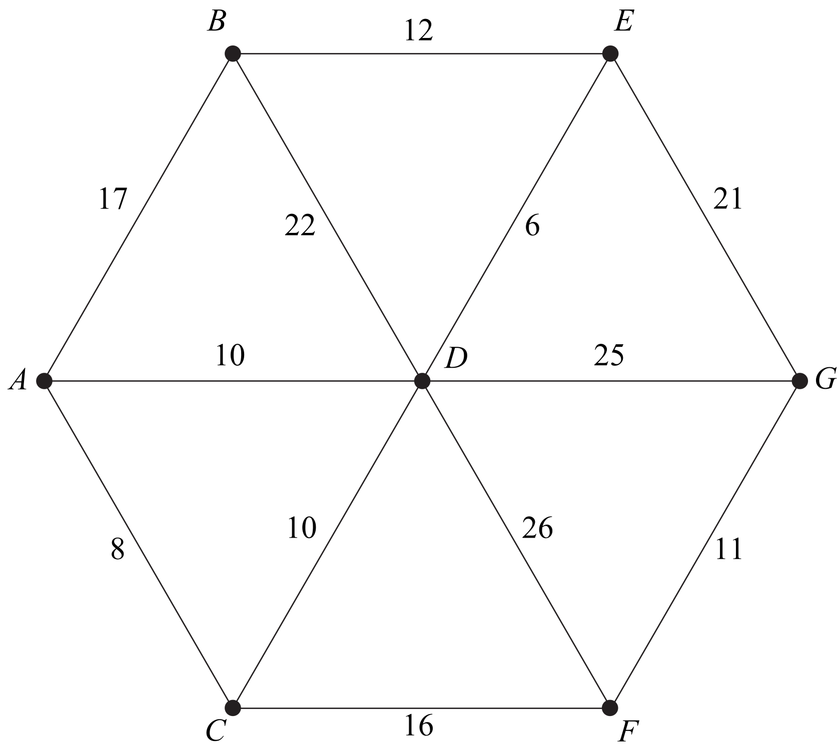
	1	**2**	**3**	**4**	**5**	**6**
A	1	1	0	0	1	1
B	0	1	0	0	1	1
C	1	0	0	0	1	1
D	0	1	1	1	0	1
E	0	0	0	0	1	1
F	0	0	0	0	0	1

(2 marks)

(b) Given that A, B, C, D, E and F represent six people and that 1, 2, 3, 4, 5 and 6 represent six tasks to which they may be assigned, explain why a complete matching is impossible. *(3 marks)*

QUESTION PART REFERENCE

3 The following network shows the roads connecting seven villages, A, B, C, ..., G. The number on each edge represents the length, in miles, between a pair of villages.



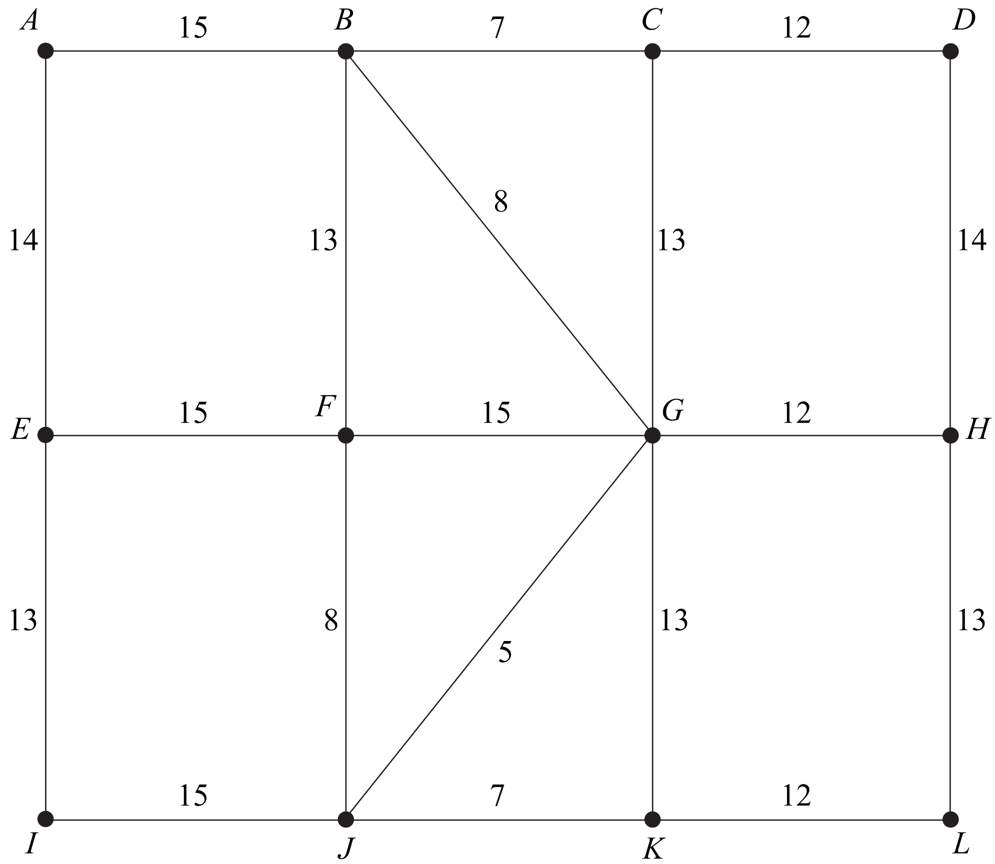
(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. *(5 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) There are two minimum spanning trees for this network. Draw both of these minimum spanning trees. *(3 marks)*

QUESTION PART REFERENCE

4 The following network shows the times, in minutes, taken by a policeman to walk along roads connecting 12 places, A, B, ..., L, on his beat. Each day, the policeman has to walk along each road at least once, starting and finishing at A.



The total of all the times in the network is 224 minutes.

(a) Find the length of an optimal Chinese postman route for the policeman. *(5 marks)*

(b) State the number of times that the vertex J would appear in a route corresponding to the length found in part **(a)**. *(1 mark)*

QUESTION PART REFERENCE

5 The feasible region of a linear programming problem is determined by the following:

$$y \geqslant 20$$
$$x + y \geqslant 25$$
$$5x + 2y \leqslant 100$$
$$y \leqslant 4x$$
$$y \geqslant 2x$$

(a) On **Figure 1** opposite, draw a suitable diagram to represent the inequalities and indicate the feasible region. *(6 marks)*

(b) Use your diagram to find the minimum value of P, on the feasible region, in the case where:

(i) $P = x + 2y$;

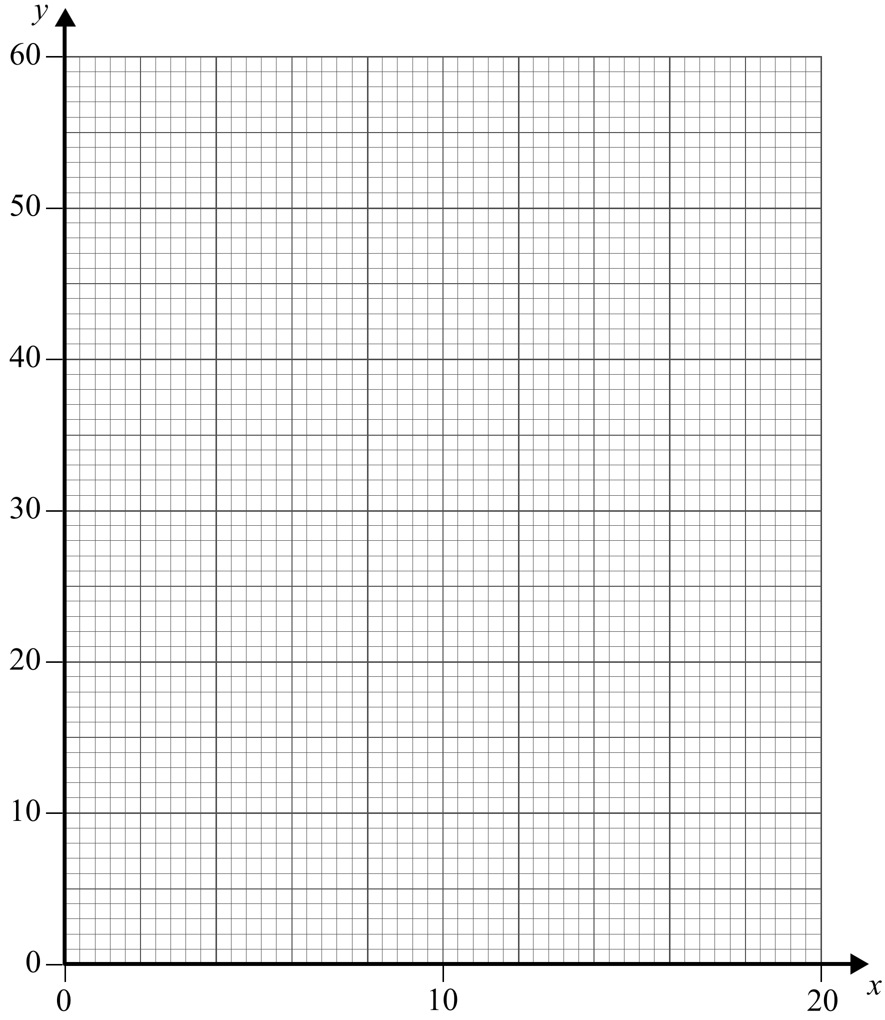
(ii) $P = -x + y$.

In each case, state the corresponding values of x and y. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Figure 1



6 The network below shows the lengths of roads, in miles, connecting 10 towns, $A, B, \ldots, J$.

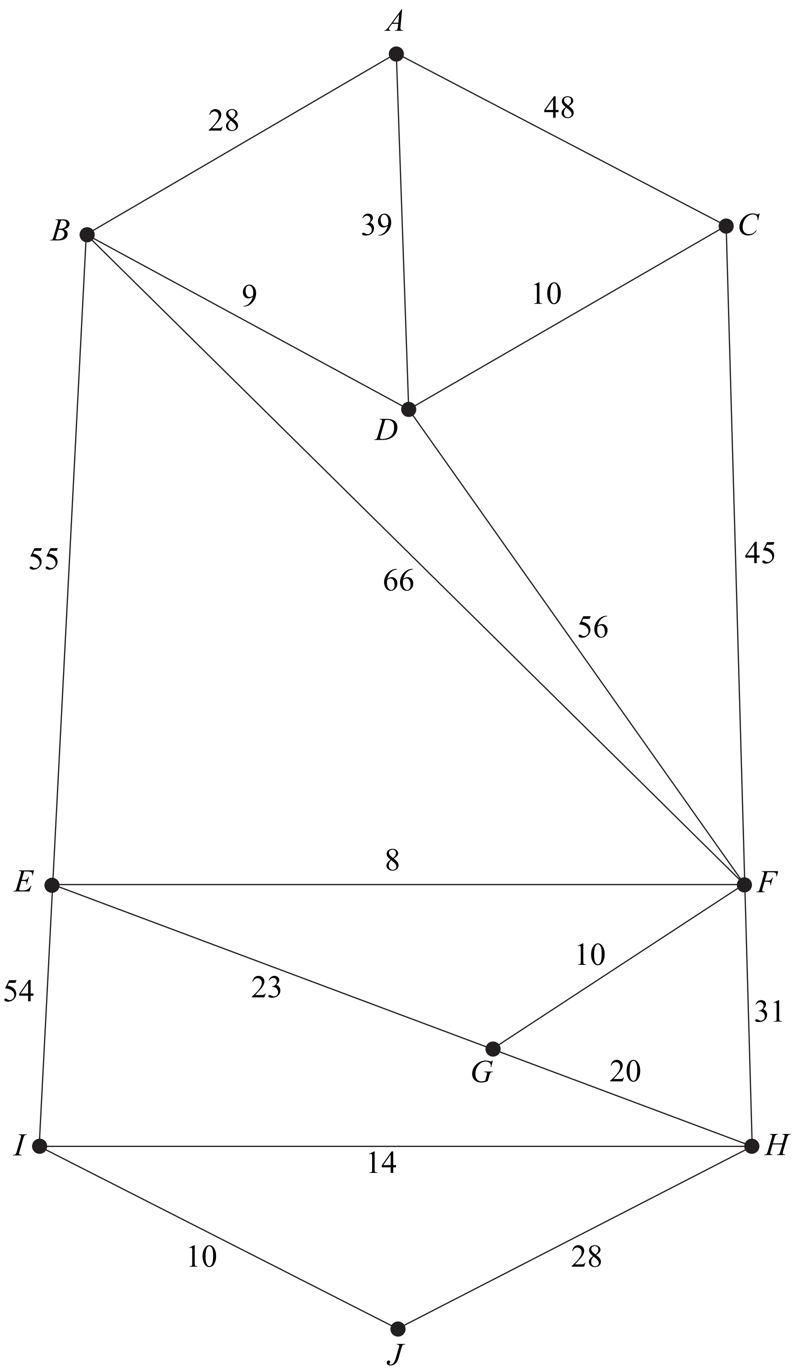
(a) Use Dijkstra's algorithm on the network to find the shortest distance from A to J. Show all your working at each vertex. *(7 marks)*

(b) Write down the corresponding route. *(1 mark)*

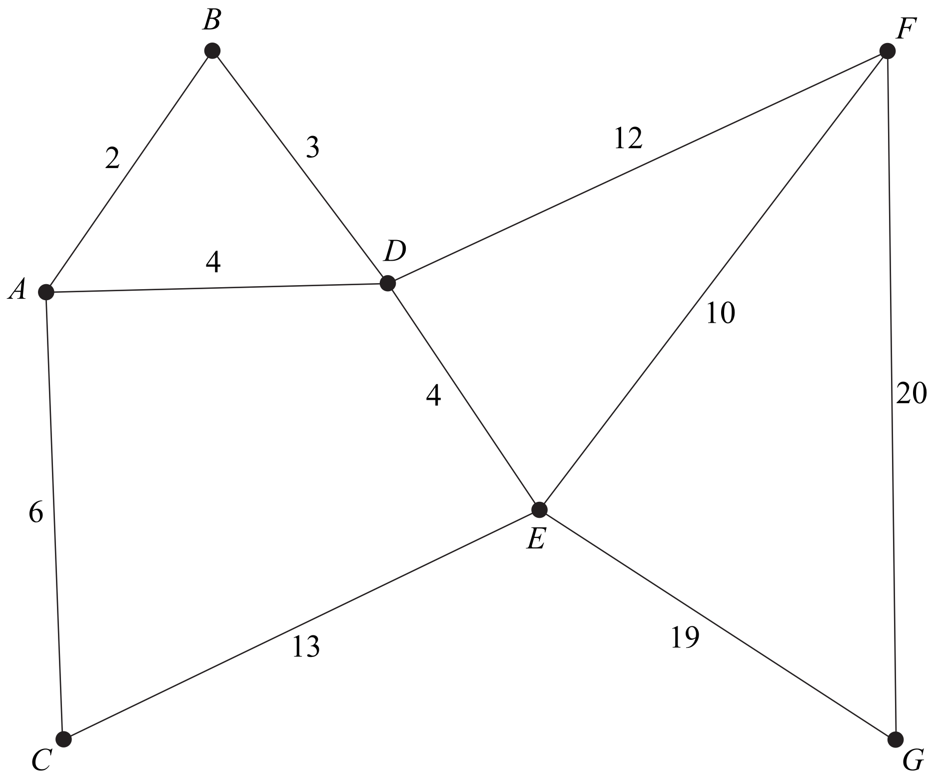
(c) A new road is to be constructed connecting B to G. Find the length of this new road if the shortest distance from A to J is reduced by 10 miles. State the new route. *(3 marks)*

QUESTION PART REFERENCE

(a)



7 The diagram shows the locations of some schools. The number on each edge shows the distance, in miles, between pairs of schools.



Sam, an adviser, intends to travel from one school to the next until he has visited all of the schools, before returning to his starting school. The shortest distances for Sam to travel between some of the schools are shown in **Table 1** opposite.

(a) Complete **Table 1**. *(2 marks)*

(b) (i) On the completed **Table 1**, use the nearest neighbour algorithm, starting from B, to find an upper bound for the length of Sam's tour. *(4 marks)*

(ii) Write down Sam's actual route if he were to follow the tour corresponding to the answer in part **(b)(i)**. *(2 marks)*

(iii) Using the nearest neighbour algorithm, starting from each of the other vertices in turn, the following upper bounds for the length of Sam's tour were obtained: 77, 77, 77, 76, 77 and 76.

Write down the best upper bound. *(1 mark)*

QUESTION PART REFERENCE

Table 1

	A	*B*	*C*	*D*	*E*	*F*	*G*
A	–	2	6	4		16	27
B	2	–	8	3		15	26
C	6	8	–	10		22	32
D	4	3	10	–		12	23
E					–		
F	16	15	22	12		–	20
G	27	26	32	23		20	–

Question 7 continues on the next page

7 (c) (i) On **Table 2** below, showing the order in which you select the edges, use Prim's algorithm, starting from A, to find a minimum spanning tree for the schools A, B, C, D, F and G. *(4 marks)*

(ii) Hence find a lower bound for the length of Sam's minimum tour. *(3 marks)*

(iii) By deleting each of the other vertices in turn, the following lower bounds for the length of a minimum tour were found: 50, 48, 52, 51, 56 and 64.

Write down the best lower bound. *(1 mark)*

(d) Given that the length of a minimum tour is T miles, use your answers to parts **(b)** and **(c)** to write down the smallest interval within which you know T must lie. *(2 marks)*

QUESTION PART REFERENCE

Table 2

	A	*B*	*C*	*D*	*F*	*G*
A	–	2	6	4	16	27
B	2	–	8	3	15	26
C	6	8	–	10	22	32
D	4	3	10	–	12	23
F	16	15	22	12	–	20
G	27	26	32	23	20	–

8 Four **distinct positive integers** are $(3x-5)$, $(2x+3)$, $(x+1)$ and $(4x-13)$.

(a) Explain why $x \geqslant 4$. *(2 marks)*

(b) The four integers are to be sorted into ascending order using a **bubble sort**.

The original list is	$(3x-5)$	$(2x+3)$	$(x+1)$	$(4x-13)$
After the first pass, the list is	$(3x-5)$	$(x+1)$	$(4x-13)$	$(2x+3)$
After the second pass, the list is	$(x+1)$	$(4x-13)$	$(3x-5)$	$(2x+3)$
After the third pass, the list is	$(4x-13)$	$(x+1)$	$(3x-5)$	$(2x+3)$

(i) By considering the list after the first pass, write down **three** inequalities in terms of x. *(3 marks)*

(ii) By considering the list after the second pass, write down **two** further inequalities in terms of x. *(2 marks)*

(iii) By considering the list after the third pass, write down **one** further inequality in terms of x. *(1 mark)*

(c) Hence, by considering the results above, find the value of x. *(2 marks)*

QUESTION PART REFERENCE

Centre Number						Candidate Number				
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Other Names										
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For Examiner's Use	
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General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MD01

Unit Decision 1

Thursday 24 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

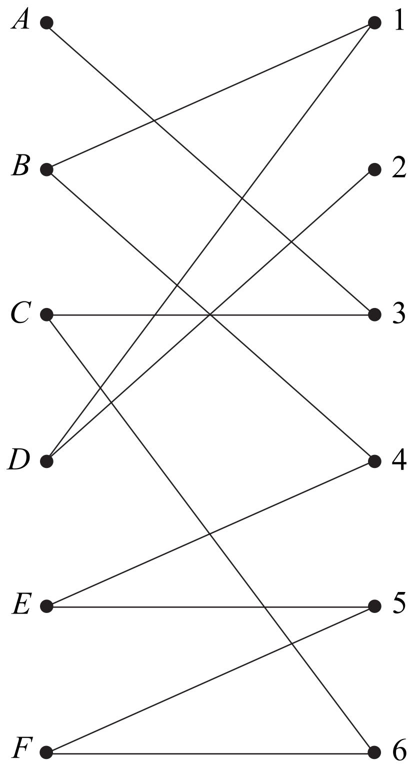
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

(b) Initially, B is assigned to task 4, C to task 3, D to task 1, E to task 5 and F to task 6.

By using an algorithm from this initial matching, find a complete matching. *(3 marks)*

QUESTION PART REFERENCE	**Answer space for question 1**

2 A student is using a shuttle sort algorithm to rearrange a set of numbers into ascending order.

Her correct solution for the first three passes is as follows.

Initial list	10	7	4	22	23	26
After 1st pass	7	10	4	22	23	26
After 2nd pass	4	7	10	22	23	26
After 3rd pass	4	7	10	22	23	26

(a) Write down the number of comparisons on each of the three passes. *(2 marks)*

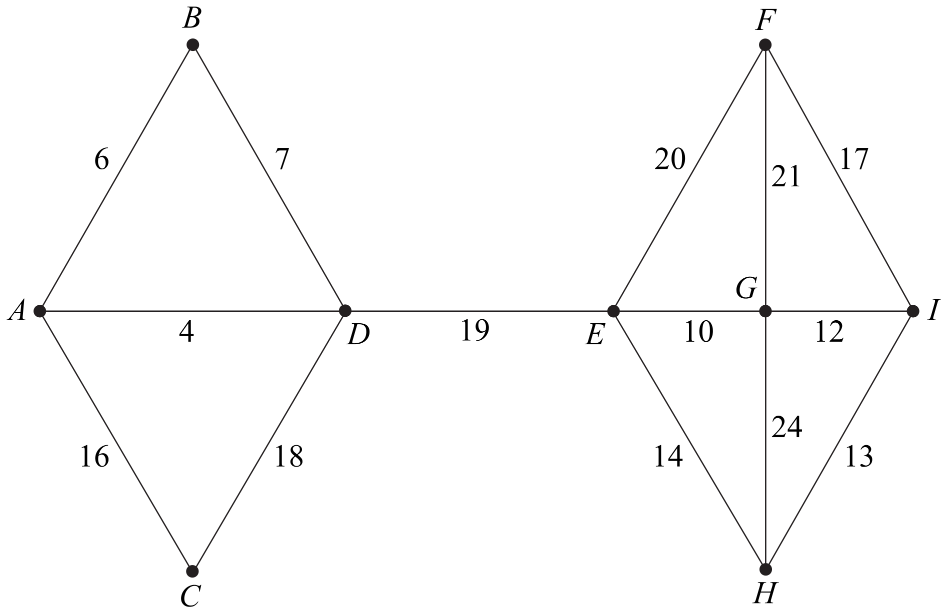
(b) Write down the number of swaps on each of the three passes. *(2 marks)*

(c) Explain whether or not the student has completed the algorithm. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

3 The following network shows the lengths, in miles, of roads connecting nine villages, A, B, …, I.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

(b) Prim's algorithm from different starting points produces the same minimum spanning tree for this network. State the final edge that would complete the minimum spanning tree using Prim's algorithm:

(i) starting from D; *(1 mark)*

(ii) starting from H. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 3

4 The edges on the network below represent some major roads in a city. The number on each edge is the minimum time taken, in minutes, to drive along that road.

(a) (i) Use Dijkstra's algorithm on the network to find the shortest possible driving time from A to J. *(5 marks)*

(ii) Write down the corresponding route. *(1 mark)*

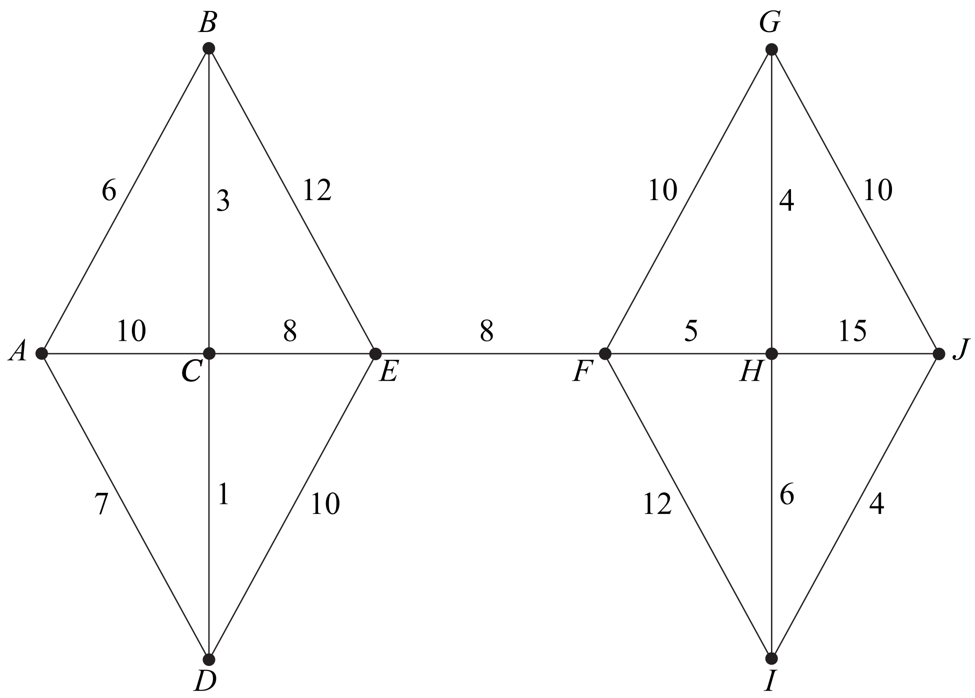
(b) A new ring road is to be constructed connecting A to J directly.

Find the maximum length of this new road from A to J if the time taken to drive along it, travelling at an average speed of 90 km/h, is to be no more than the time found in part **(a)(i)**. *(2 marks)*

QUESTION PART REFERENCE

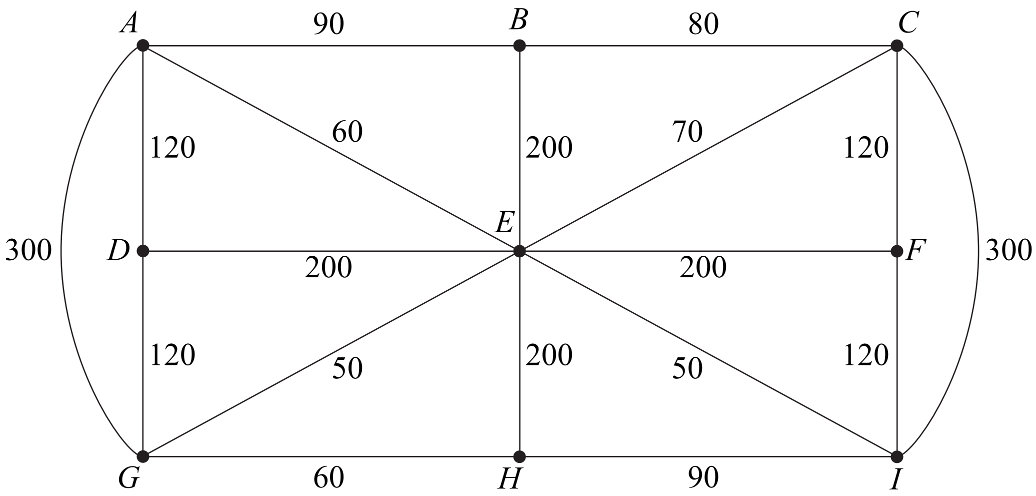
Answer space for question 4

(a)(i)



5 The network below shows some streets in a town. The number on each edge shows the length of that street, in metres.

Leaflets are to be distributed by a restaurant owner, Tony, from his restaurant located at vertex B. Tony must start from his restaurant, walk along all the streets at least once, before returning to his restaurant.



The total length of the streets is 2430 metres.

(a) Find the length of an optimal Chinese postman route for Tony. *(5 marks)*

(b) Colin also wishes to distribute some leaflets. He starts from his house at H, walks along all the streets at least once, before finishing at the restaurant at B.

Colin wishes to walk the minimum distance. Find the length of an optimal route for Colin. *(1 mark)*

(c) David also walks along all the streets at least once. He can start at any vertex and finish at any vertex. David also wishes to walk the minimum distance.

(i) Find the length of an optimal route for David. *(1 mark)*

(ii) State the vertices from which David could start in order to achieve this optimal route. *(1 mark)*

QUESTION PART REFERENCE	Answer space for question 5

6 The complete graph K_n $(n > 1)$ has every one of its n vertices connected to each of the other vertices by a single edge.

(a) Draw the complete graph K_4. *(1 mark)*

(b) (i) Find the total number of edges for the graph K_8.

(ii) Give a reason why K_8 is not Eulerian. *(2 marks)*

(c) For the graph K_n, state in terms of n:

(i) the total number of edges;

(ii) the number of edges in a minimum spanning tree;

(iii) the condition for K_n to be Eulerian;

(iv) the condition for the number of edges of a Hamiltonian cycle to be equal to the number of edges of an Eulerian cycle. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 6

7 Rupta, a sales representative, has to visit six shops, A, B, C, D, E and F. Rupta starts at shop A and travels to each of the other shops once, before returning to shop A. Rupta wishes to keep her travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	A	***B***	***C***	***D***	***E***	***F***
A	–	16	10	25	26	40
B	16	–	20	19	18	50
C	10	20	–	14	22	31
D	25	19	14	–	11	32
E	26	18	22	11	–	42
F	40	50	31	32	42	–

(a) Find the travelling time of the tour $ACFDEBA$. *(1 mark)*

(b) Use the nearest neighbour algorithm, starting at A, to find an upper bound for the travelling time for Rupta's tour. *(4 marks)*

(c) By deleting A, find a lower bound for the travelling time for Rupta's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

8 The following algorithm finds an estimate of the value of the number represented by the symbol e:

Line 10	Let $A = 1$, $B = 1$, $C = 1$
Line 20	Let $D = A$
Line 30	Let $C = C \times B$
Line 40	Let $D = D + (1/C)$
Line 50	If $B = 4$ then go to Line 80
Line 60	Let $B = B + 1$
Line 70	Go to Line 30
Line 80	Print 'An estimate of e is', D
Line 90	End

(a) Trace the algorithm. *(6 marks)*

(b) A student miscopied Line 70.

His line was

Line 70 Go to Line 10

Explain what would happen if his algorithm were traced. *(2 marks)*

QUESTION PART REFERENCE	**Answer space for question 8**

Do not write outside the box

9 Ollyin is buying new pillows for his hotel. He buys three types of pillow: soft, medium and firm.

He must buy at least 100 soft pillows and at least 200 medium pillows.

He must buy at least 400 pillows in total.

Soft pillows cost £4 each. Medium pillows cost £3 each. Firm pillows cost £4 each.

He wishes to spend no more than £1800 on new pillows.

At least 40% of the new pillows must be medium pillows.

Ollyin buys x soft pillows, y medium pillows and z firm pillows.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find five inequalities in x, y and z that model the above constraints. *(3 marks)*

(b) Ollyin decides to buy twice as many soft pillows as firm pillows.

(i) Show that three of your answers in part **(a)** become

$$3x + 2y \geqslant 800$$
$$2x + y \leqslant 600$$
$$y \geqslant x$$

(3 marks)

(ii) On the grid opposite, draw a suitable diagram to represent Ollyin's situation, indicating the feasible region. *(5 marks)*

(iii) Use your diagram to find the maximum total number of pillows that Ollyin can buy. *(2 marks)*

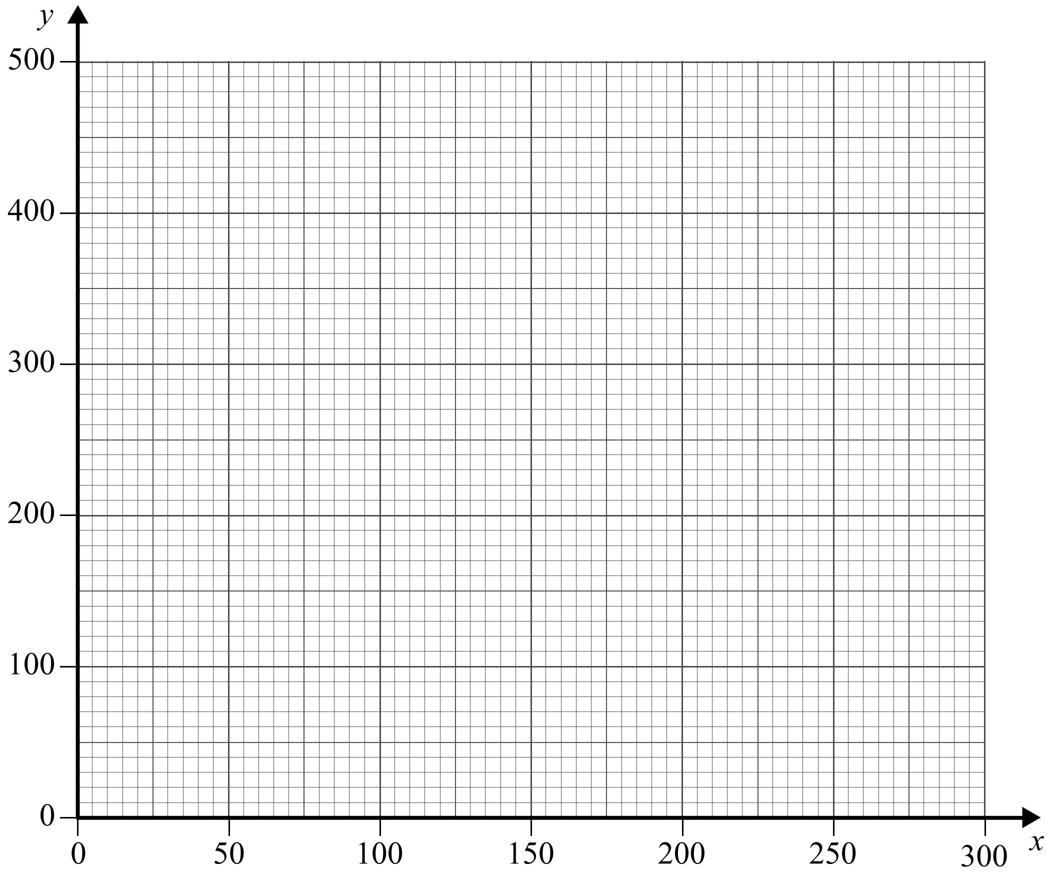
(iv) Find the number of each type of pillow that Ollyin can buy that corresponds to your answer to part **(b)(iii)**. *(1 mark)*

QUESTION PART REFERENCE	**Answer space for question 9**

QUESTION PART REFERENCE

Answer space for question 9

(b)(ii)



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AQA

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MD01

Unit Decision 1

Friday 25 January 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Draw a bipartite graph to represent the following adjacency matrix.

	1	**2**	**3**	**4**	**5**
A	0	0	0	0	1
B	0	0	0	1	1
C	0	1	0	1	1
D	0	0	0	1	1
E	1	1	1	0	1

(2 marks)

(b) If A, B, C, D and E represent five people and 1, 2, 3, 4 and 5 represent five tasks to which they are to be assigned, explain why a complete matching is impossible.

(2 marks)

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

Turn over ▶

2 (a) Use a Shell sort to arrange the following numbers into ascending order.

7 8 1 6 3 4 5 2 *(4 marks)*

(b) Write down the number of comparisons on the first pass. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

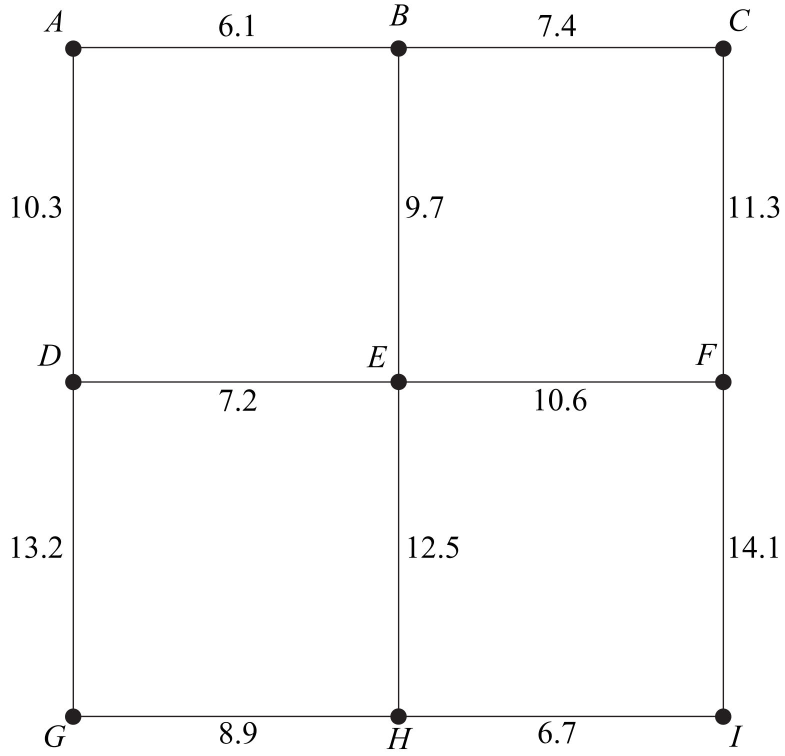
QUESTION PART REFERENCE

Answer space for question 2

Turn over ▶

3 The following network shows the lengths, in miles, of roads connecting nine villages, A, B, …, I.

A delivery man lives in village A and is to drive along all the roads at least once before returning to A.



Total length of all the roads is 118 miles

(a) Find the length of an optimal Chinese postman route around the nine villages, starting and finishing at A. *(5 marks)*

(b) For an optimal Chinese postman route corresponding to your answer in part **(a)**, state:

(i) the number of times village E would be visited;

(ii) the number of times village I would be visited. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

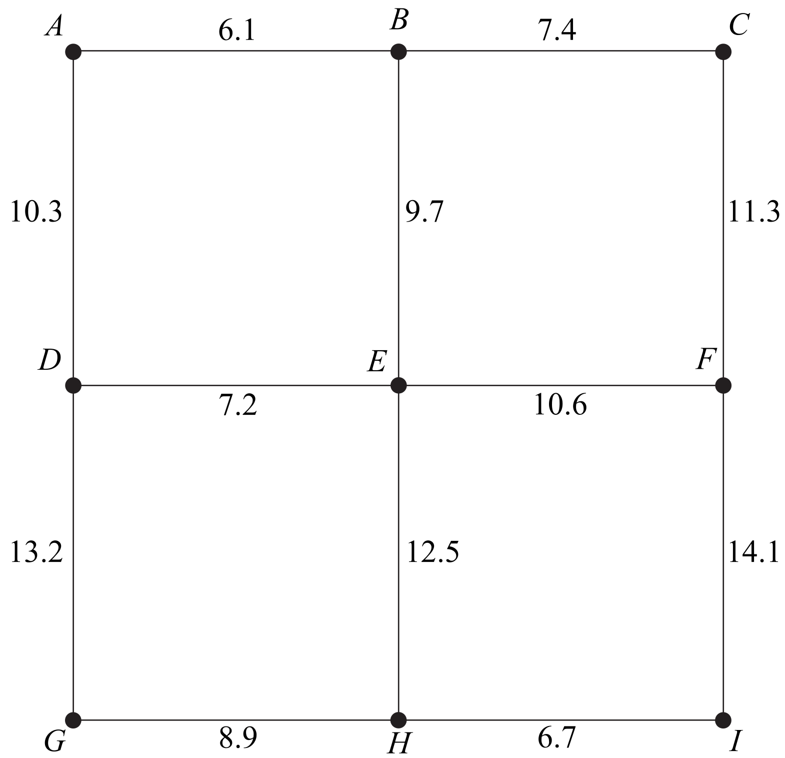
QUESTION PART REFERENCE

Answer space for question 3

Turn over ▶

4 The following network shows the lengths, in miles, of roads connecting nine villages, A, B, …, I.

A programme of resurfacing some roads is undertaken to ensure that each village can access all other villages along a resurfaced road, while keeping the amount of road to be resurfaced to a minimum.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network.

(ii) State the length of your minimum spanning tree.

(iii) Draw your minimum spanning tree. *(7 marks)*

(b) Given that Prim's algorithm is used with different start vertices, state the final edge to be added to the minimum spanning tree if:

(i) the start vertex is E;

(ii) the start vertex is G. *(2 marks)*

(c) Given that Kruskal's algorithm is used to find the minimum spanning tree, state which edge would be:

(i) the first to be included in the tree;

(ii) the last to be included in the tree. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 4

5 The feasible region of a linear programming problem is defined by

$$x + y \leqslant 60$$
$$2x + y \leqslant 80$$
$$y \geqslant 20$$
$$x \geqslant 15$$
$$y \geqslant x$$

(a) On the grid opposite, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) In each of the following cases, use your diagram to find the maximum value of P on the feasible region. In each case, state the corresponding values of x and y.

(i) $P = x + 4y$ *(2 marks)*

(ii) $P = 4x + y$ *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

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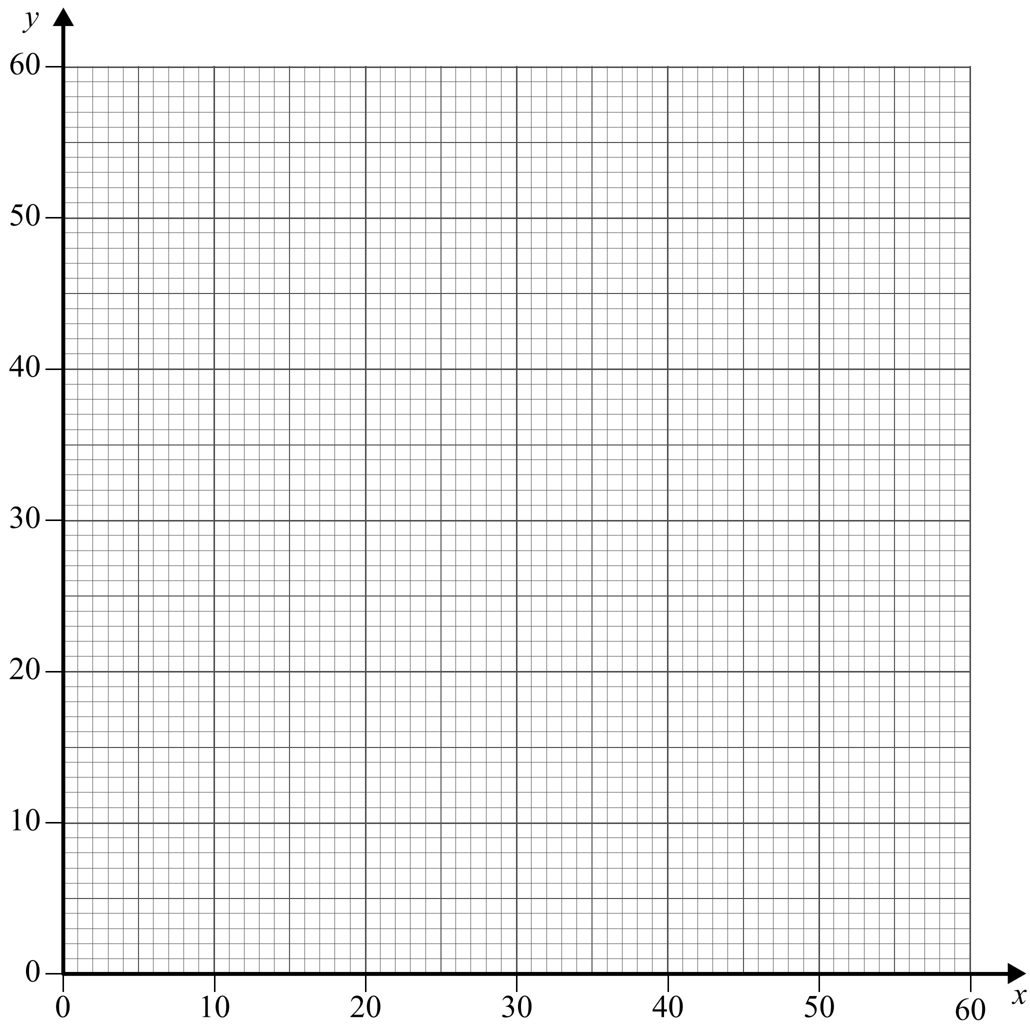
QUESTION PART REFERENCE

Answer space for question 5

(a)



6 The network opposite shows some roads connecting towns. The number on each edge represents the length, in miles, of the road connecting a pair of towns.

(a) (i) Use Dijkstra's algorithm on the network to find the minimum distance from A to J.

(ii) Write down the corresponding route. *(7 marks)*

(b) The road AJ is a dual carriageway. Ken drives at 60 miles per hour on this road and drives at 50 miles per hour on all other roads.

Find the minimum time to travel from A to J. *(3 marks)*

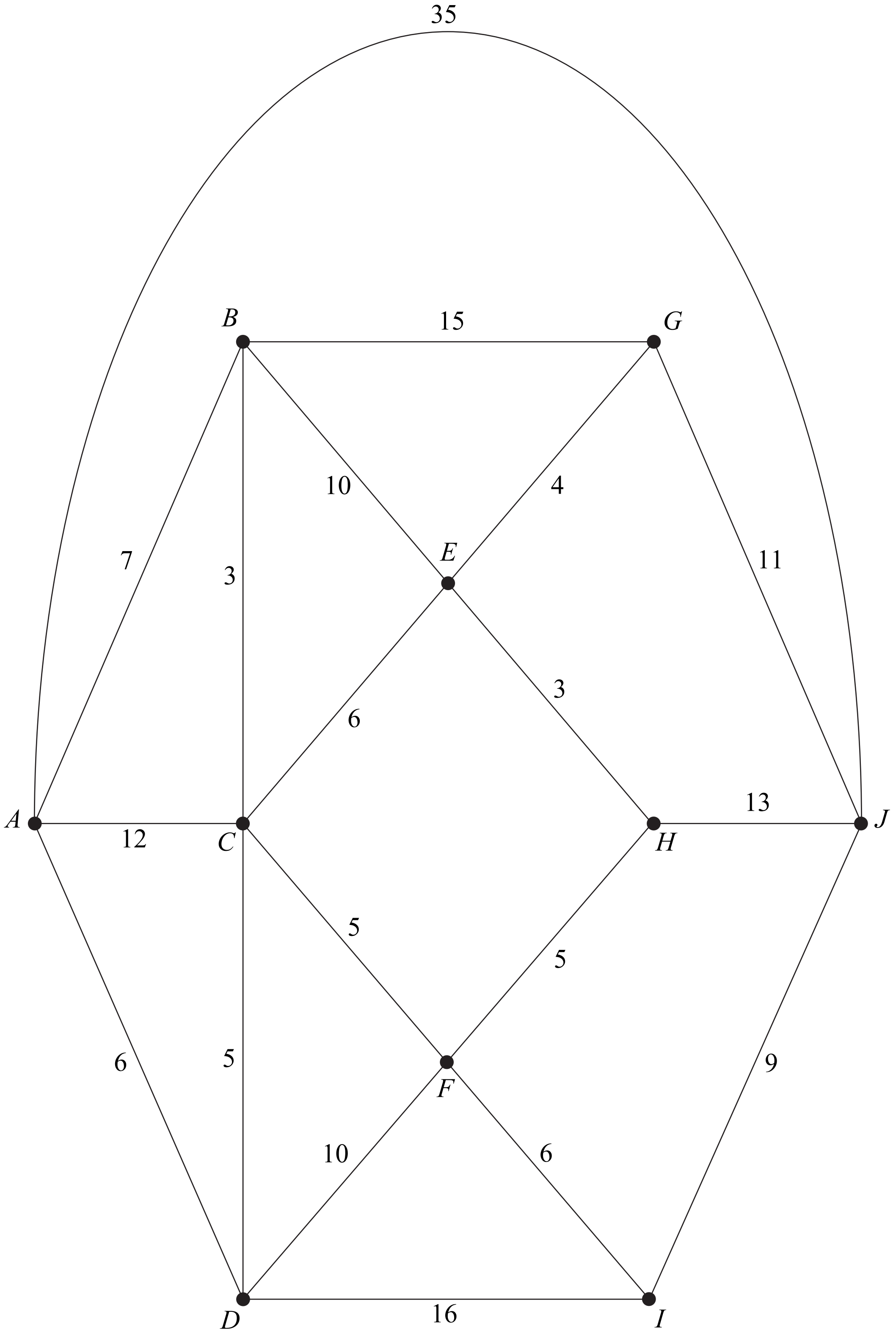
QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

(a)



Turn over ▶

7 (a) A simple connected graph X has eight vertices.

(i) State the minimum number of edges of the graph.

(ii) Find the maximum number of edges of the graph. *(2 marks)*

(b) A simple connected graph Y has n vertices.

(i) State the minimum number of edges of the graph.

(ii) Find the maximum number of edges of the graph. *(2 marks)*

(c) A simple graph Z has six vertices and each of the vertices has the same degree d.

(i) State the possible values of d.

(ii) If Z is connected, state the possible values of d.

(iii) If Z is Eulerian, state the possible values of d. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 7

8 Tony delivers paper to five offices, A, B, C, D and E. Tony starts his deliveries at office E and travels to each of the other offices once, before returning to office E. Tony wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the offices.

	A	***B***	***C***	***D***	***E***
A	–	10	16	20	8
B	10	–	21	15	9
C	16	21	–	10	23
D	20	15	10	–	17
E	8	9	23	17	–

(a) Find the travelling time of the tour $ACDBEA$. *(1 mark)*

(b) Hence write down a tour, starting at E, which has the same total travelling time as your answer to part **(a)**. *(1 mark)*

(c) Use the nearest neighbour algorithm, starting at E, to find an upper bound for the minimum travelling time for Tony's tour. *(4 marks)*

(d) By deleting E, find a lower bound for the minimum travelling time for Tony's tour. *(4 marks)*

(e) Sketch a network showing the edges that give the lower bound in part **(d)**, and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

9 A factory can make three different kinds of balloon pack: gold, silver and bronze. Each pack contains three different types of balloon: A, B and C.

Each gold pack has 2 type A balloons, 3 type B balloons and 6 type C balloons.

Each silver pack has 3 type A balloons, 4 type B balloons and 2 type C balloons.

Each bronze pack has 5 type A balloons, 3 type B balloons and 2 type C balloons.

Every hour, the maximum number of each type of balloon available is 400 type A, 400 type B and 400 type C.

Every hour, the factory must pack at least 1000 balloons.

Every hour, the factory must pack more type A balloons than type B balloons.

Every hour, the factory must ensure that no more than 40% of the total balloons packed are type C balloons.

Every hour, the factory makes x gold, y silver and z bronze packs.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, simplifying your answers. *(8 marks)*

QUESTION PART REFERENCE

Answer space for question 9

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

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General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MD01

Unit Decision 1

Friday 24 May 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

J U N 1 3 M D 0 1 0 1

P61146/Jun13/MD01 6/6/6/

Answer **all** questions.

Answer each question in the space provided for that question.

1 Six people, Andy, Bob, Colin, Dev, Eric and Faisal, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following table shows the tasks that each person is able to undertake.

Person	Task
Andy	1, 3
Bob	1, 4
Colin	2, 3
Dev	4, 5, 6
Eric	2, 5, 6
Faisal	1, 3

(a) Represent this information on a bipartite graph. *(2 marks)*

(b) Initially, Bob is allocated to task 1, Colin to task 3, Dev to task 5 and Eric to task 2.

Demonstrate, by using an alternating path algorithm from this initial matching, how each person can be allocated to a different task. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 1

2 (a) Use the quicksort algorithm to rearrange the following numbers into ascending order, showing the new arrangement after each pass. You must indicate the pivot(s) being used on each pass.

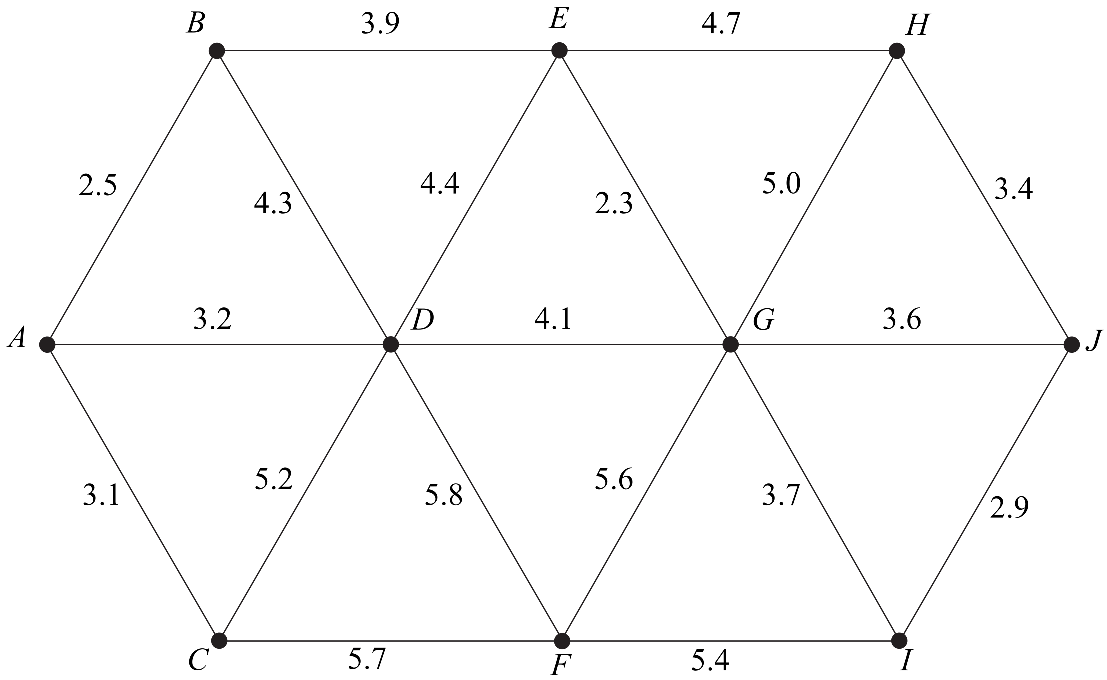
2, 12, 17, 18, 5, 13 *(4 marks)*

(b) For the **first** pass, write down the number of comparisons. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

3 The following network shows the lengths, in miles, of roads connecting ten villages, $A, B, C, \ldots, J$.



(a) (i) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the network.

(ii) Find the length of your minimum spanning tree.

(iii) Draw your minimum spanning tree. *(7 marks)*

(b) Prim's algorithm from different starting points produces the same minimum spanning tree. State the final edge that would be added to complete the minimum spanning tree if the starting point were:

(i) A;

(ii) F. *(2 marks)*

QUESTION PART REFERENCE **Answer space for question 3**

4 Sarah is a mobile hairdresser based at A. Her day's appointments are at five places: B, C, D, E and F. She can arrange the appointments in any order. She intends to travel from one place to the next until she has visited all of the places, starting and finishing at A. The following table shows the times, in minutes, that it takes to travel between the six places.

	A	***B***	***C***	***D***	***E***	***F***
A	–	15	11	14	27	12
B	15	–	13	19	24	15
C	11	13	–	10	19	12
D	14	19	10	–	26	15
E	27	24	19	26	–	27
F	12	15	12	15	27	–

(a) Sarah decides to visit the places in the order $ABCDEFA$. Find the travelling time of this tour. *(1 mark)*

(b) Explain why this answer can be considered as being an upper bound for the minimum travelling time of Sarah's tour. *(2 marks)*

(c) Use the nearest neighbour algorithm, starting from A, to find another upper bound for the minimum travelling time of Sarah's tour. *(4 marks)*

(d) By deleting A, find a lower bound for the minimum travelling time of Sarah's tour. *(4 marks)*

(e) Sarah thinks that she can reduce her travelling time to 75 minutes. Explain why she is wrong. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

5 The network on the page opposite shows the times, in minutes, taken by police cars to drive along roads connecting 12 places, A, B, …, L.

On a particular day, there are three police cars in the area at A, E and J. There is an emergency at G and all three police cars drive to G.

(a) (i) Use Dijkstra's algorithm on the network, **starting from *G***, to find the minimum driving time for each of the police cars to arrive at G. *(7 marks)*

(ii) For each of the police cars, write down the route corresponding to the minimum driving time in your answer to part **(a)(i)**. *(3 marks)*

(b) Each day, a police car has to drive along each road at least once, starting and finishing at A.

For an optimal Chinese postman route:

(i) find the driving time for the police car; *(6 marks)*

(ii) state the number of times that the vertex B would appear. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 5

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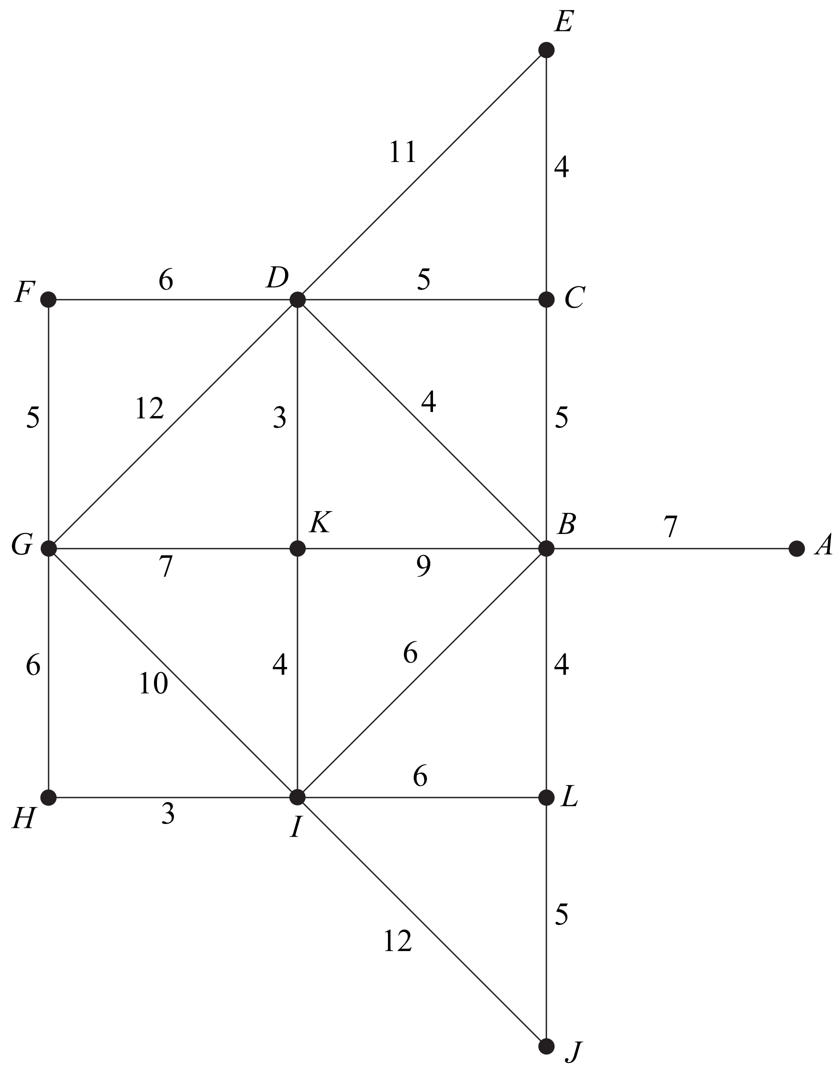
QUESTION PART REFERENCE

Answer space for question 5



Total of all times = 134 minutes

Turn over ▶

6 A student is tracing the following algorithm. The function INT gives the integer part of any number, eg $\text{INT}(2.3) = 2$ and $\text{INT}(6.7) = 6$.

Line 10	Input A, B
Line 20	Let $C = \text{INT}(A \div B)$
Line 30	Let $D = B \times C$
Line 40	Let $E = A - D$
Line 50	If $E = 0$ then go to Line 90
Line 60	Let $A = B$
Line 70	Let $B = E$
Line 80	Go to Line 20
Line 90	Print B
Line 100	Stop

(a) Trace the algorithm when the input values are:

(i) $A = 36$ and $B = 16$; *(3 marks)*

(ii) $A = 11$ and $B = 7$. *(5 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 6

7 Paul is a florist. Every day, he makes three types of floral bouquet: gold, silver and bronze.

Each gold bouquet has 6 roses, 6 carnations and 6 dahlias.

Each silver bouquet has 4 roses, 6 carnations and 4 dahlias.

Each bronze bouquet has 3 roses, 4 carnations and 4 dahlias.

Each day, Paul must use at least 420 roses and at least 480 carnations, but he can use at most 720 dahlias.

Each day, Paul makes x gold bouquets, y silver bouquets and z bronze bouquets.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find three inequalities in x, y and z that model the above constraints. *(3 marks)*

(b) On a particular day, Paul makes the same number of silver bouquets as bronze bouquets.

(i) Show that x and y must satisfy the following inequalities.

$$6x + 7y \geqslant 420$$
$$3x + 5y \geqslant 240$$
$$3x + 4y \leqslant 360$$

(2 marks)

(ii) Paul makes a profit of £4 on each gold bouquet sold, a profit of £2.50 on each silver bouquet sold and a profit of £2.50 on each bronze bouquet sold. Each day, Paul sells all the bouquets he makes. Paul wishes to maximise his daily profit, £P.

Draw a suitable diagram, on the grid opposite, to enable this problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*

(iii) Use your diagram to find Paul's maximum daily profit and the number of each type of bouquet he must make to achieve this maximum. *(2 marks)*

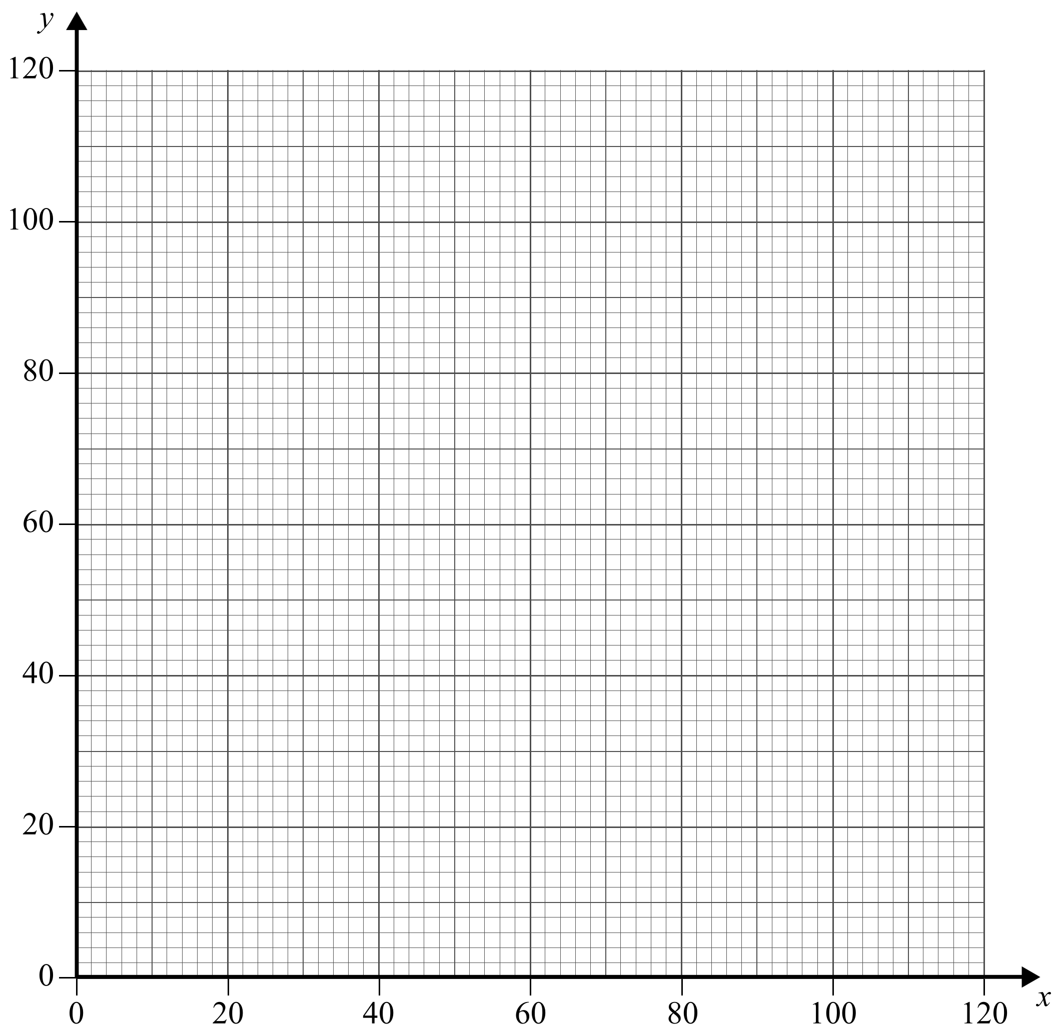
(c) On another day, Paul again makes the same number of silver bouquets as bronze bouquets, but he makes a profit of £2 on each gold bouquet sold, a profit of £6 on each silver bouquet sold and a profit of £6 on each bronze bouquet sold.

Find Paul's maximum daily profit, and the number of each type of bouquet he must make to achieve this maximum. *(3 marks)*

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7



Turn over ▶

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics MD01

Unit Decision 1

Wednesday 18 June 2014 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
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Information
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- The maximum mark for this paper is 75.

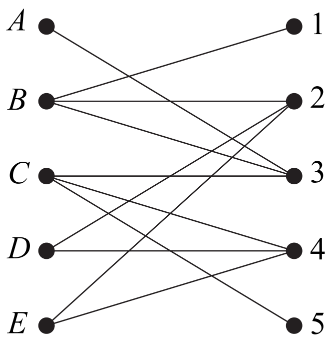
Advice
- You do not necessarily need to use all the space provided.

P71804/Jun14/E7w

Answer **all** questions.

Answer each question in the space provided for that question.

1 Five people, A, B, C, D and E, are to be allocated to five tasks, 1, 2, 3, 4 and 5.
The following bipartite graph shows the tasks that each person is able to undertake.



(a) Represent this information in an adjacency matrix.

[2 marks]

(b) Initially, A is allocated to task 3, B to task 2 and C to task 4.

(i) Demonstrate, by using an alternating-path algorithm from this initial matching, how each person can be allocated to a different task.

[3 marks]

(ii) Find a different allocation of people to tasks.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

2 A document which is currently written in English is to be translated into six other European Union languages. The cost of translating a document varies, as it is harder to find translators for some languages.

The costs, in euros, are shown in the table below.

(a) (i) On the **table below**, showing the order in which you select the edges, use Prim's algorithm, starting from E, to find a minimum spanning tree for the graph connecting D, E, F, G, H, I and S.

[5 marks]

(ii) Find the length of your minimum spanning tree.

[1 mark]

(iii) Draw your minimum spanning tree.

[2 marks]

(b) It is given that the graph has a unique minimum spanning tree.

State the final two edges that would be added to complete the minimum spanning tree in the case where:

(i) Prim's algorithm starting from H is used;

(ii) Kruskal's algorithm is used.

[3 marks]

Answer space for question 2

	Danish (D)	**English (E)**	**French (F)**	**German (G)**	**Hungarian (H)**	**Italian (I)**	**Spanish (S)**
Danish (D)	–	120	140	80	170	140	140
English (E)	120	–	70	80	130	130	110
French (F)	140	70	–	90	190	85	90
German (G)	80	80	90	–	110	100	100
Hungarian (H)	170	130	190	110	–	140	150
Italian (I)	140	130	85	100	140	–	60
Spanish (S)	140	110	90	100	150	60	–

3 The network below shows 11 towns, A, B, ..., K. The number on each edge shows the time, in minutes, to travel between a pair of towns.

(a) (i) Use Dijkstra's algorithm on the diagram below to find the minimum time to travel from A to K.

[6 marks]

(ii) State the corresponding route.

[1 mark]

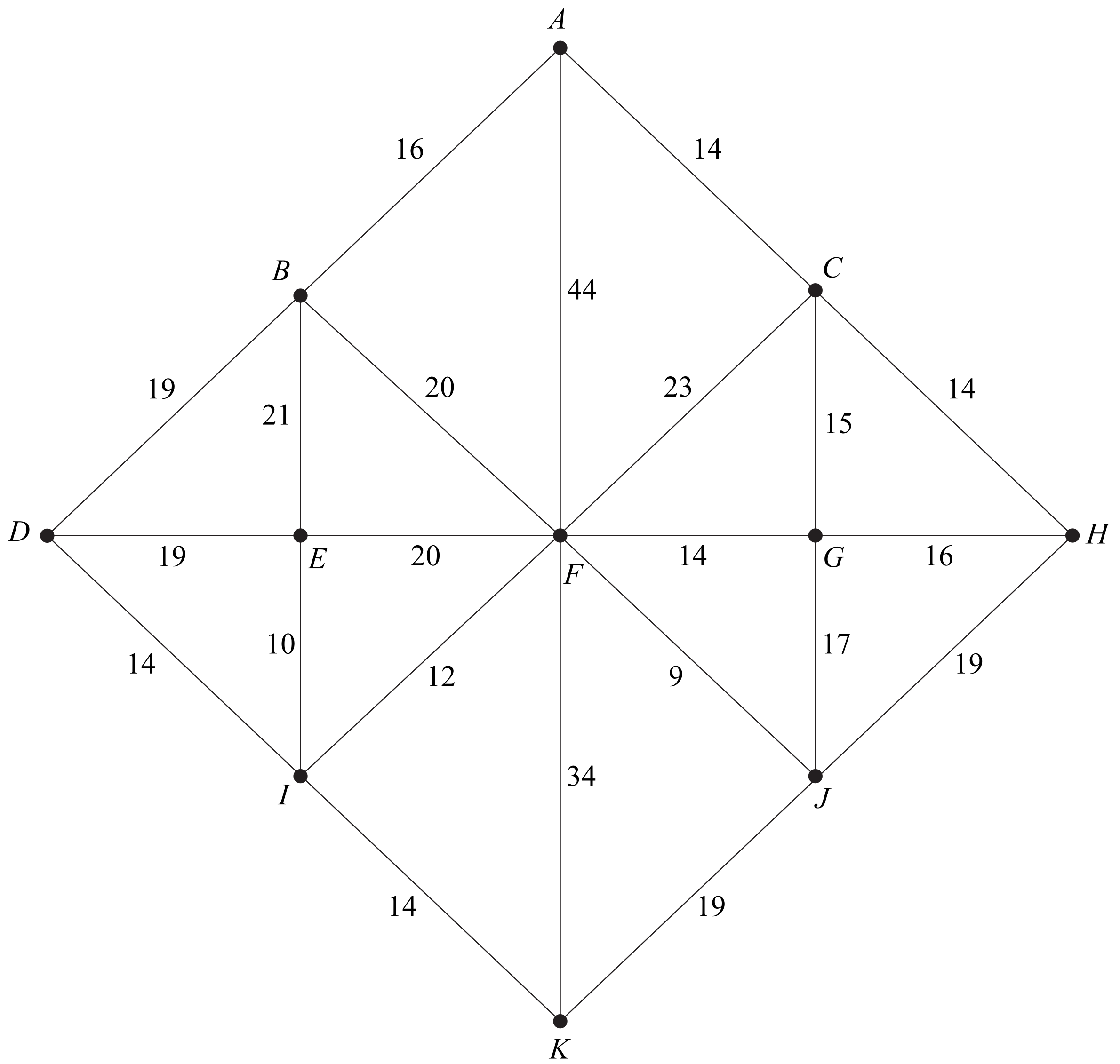
(b) On a particular day, Jenny travels from A to K but visits her friend at D on her way. Find Jenny's minimum travelling time.

[1 mark]

(c) On a different day, all roads connected to I are closed due to flooding. Jenny does not visit her friend at D. Find her minimum time to travel from A to K. State the route corresponding to this minimum time.

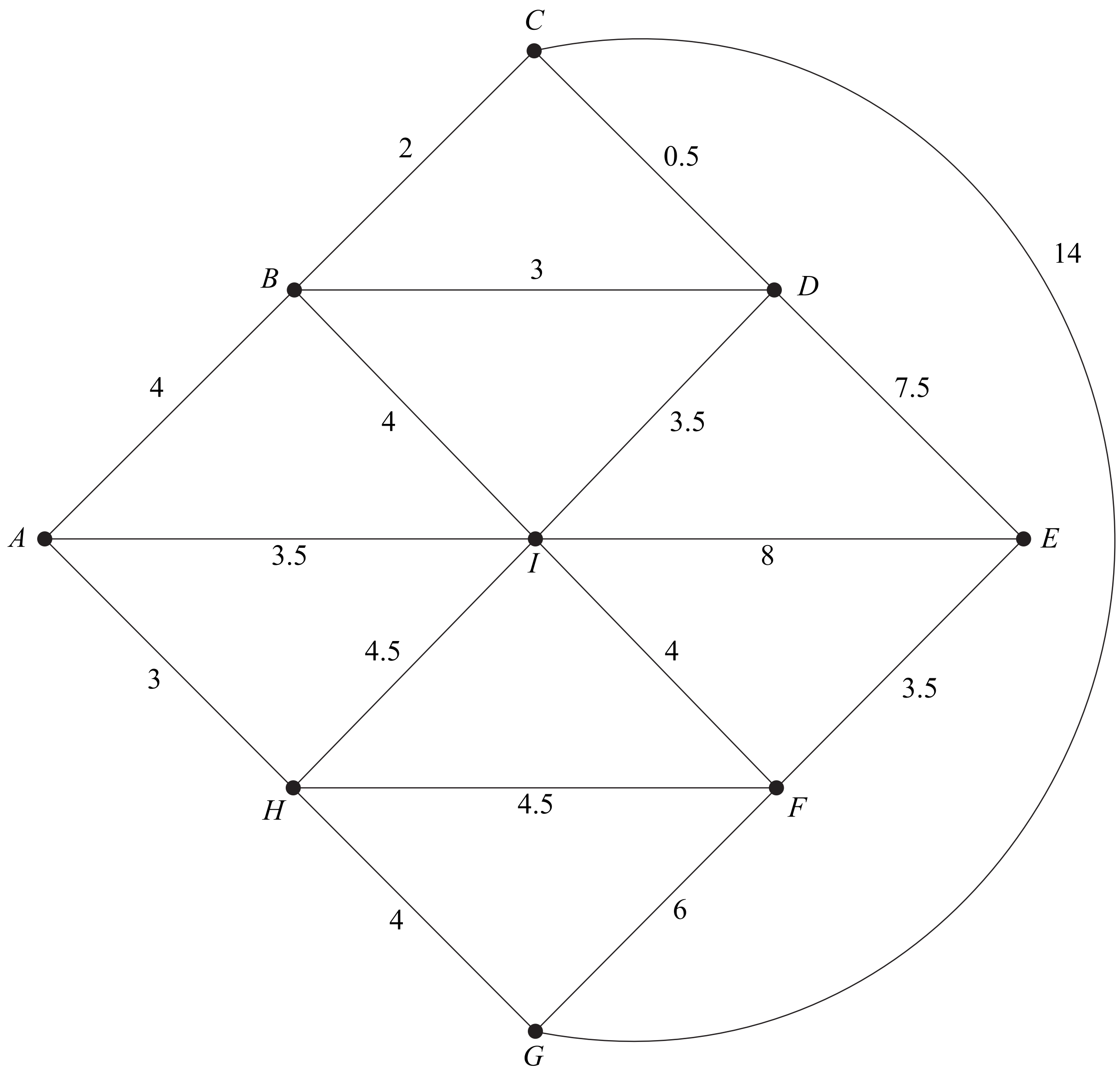
[2 marks]

Answer space for question 3



4 Paulo sells vegetables from his van. He drives around the streets of a small village. The network shows the streets in the village. The number on each edge shows the time, in minutes, to drive along that street.

Paulo starts from his house located at vertex A and drives along all the streets at least once before returning to his house.



The total of all the times in the diagram is 79.5 minutes.

(a) Find the length of an optimal Chinese postman route around the village, starting and finishing at A. (Shortest routes between vertices may be found by inspection.)

[5 marks]

(b) For an optimal Chinese postman route, state:

(i) the number of times the vertex F would occur;

(ii) the number of times the vertex D would occur.

[2 marks]

(c) Toto is standing for the position of Mayor in the local elections. He intends to travel along all the roads at least once. He can start his journey at any vertex and can finish his journey at any vertex.

(i) Find the length of an optimal route for Toto.

(ii) State the vertices from which Toto could start in order to achieve this optimal route.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

5 The feasible region of a linear programming problem is determined by the following:

$$x \geqslant 1$$
$$y \geqslant 3$$
$$x + y \geqslant 5$$
$$x + y \leqslant 12$$
$$3x + 8y \leqslant 64$$

(a) On the grid below, draw a suitable diagram to represent the inequalities and indicate the feasible region.

[5 marks]

(b) Use your diagram to find, on the feasible region:

(i) the maximum value of $3x + y$;

(ii) the maximum value of $2x + 3y$;

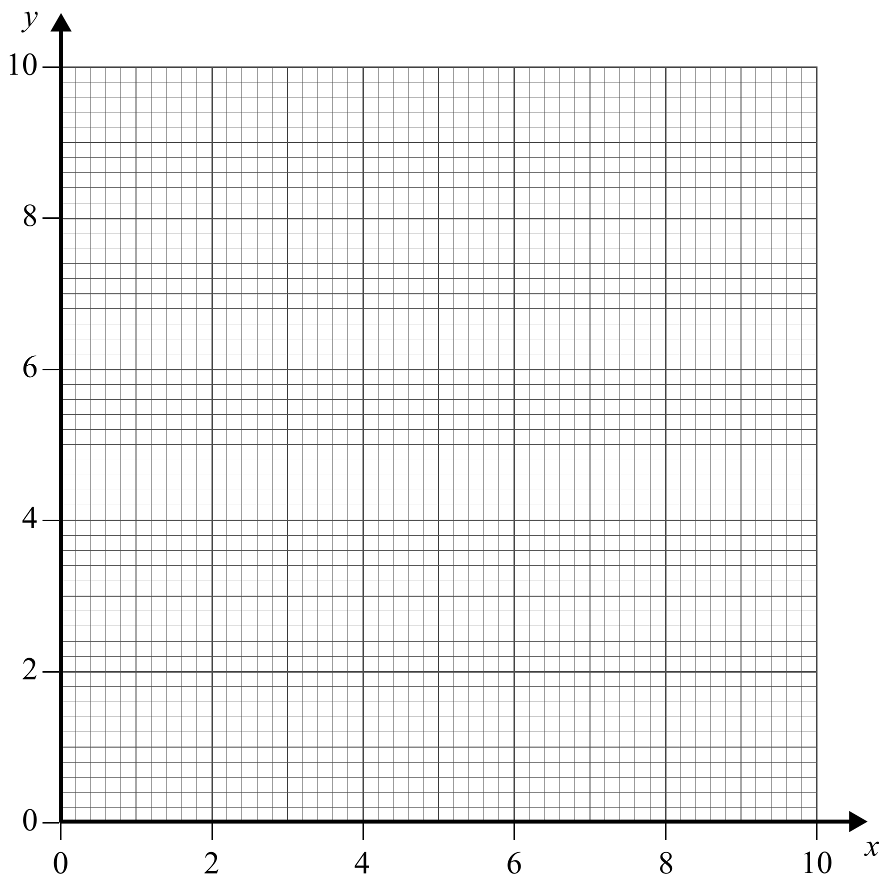
(iii) the minimum value of $-2x + y$.

In each case, state the coordinates of the point corresponding to your answer.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 5



6 (a) Sarah is solving a travelling-salesman problem.

(i) She finds the following upper bounds: 32, 33, 32, 32, 30, 32, 32.

Write down the best upper bound.

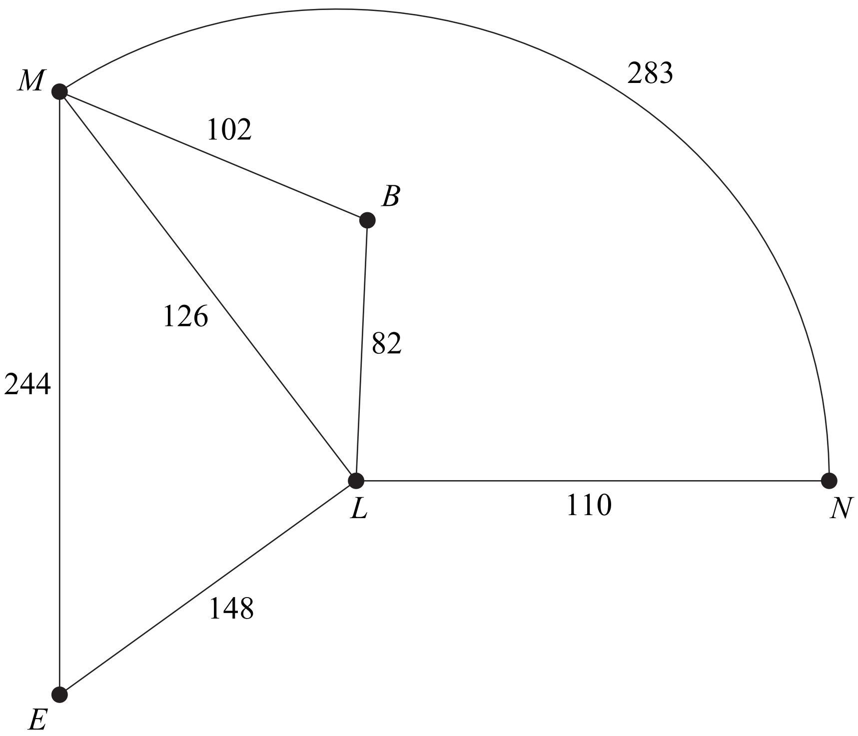
(ii) She finds the following lower bounds: 17, 18, 17, 20, 18, 17, 20.

Write down the best lower bound.

[2 marks]

(b) Rob is travelling by train to a number of cities. He is to start at M and visit each other city at least once before returning to M.

The diagram shows the travelling times, in minutes, between cities. Where no time is shown, there is no direct journey available.



The table below shows the minimum travelling times between all pairs of cities.

	B	***E***	***L***	***M***	***N***
B	–	230	82	102	192
E	230	–	148	244	258
L	82	148	–	126	110
M	102	244	126	–	236
N	192	258	110	236	–

(i) Explain why the minimum travelling time from M to N is not 283.
[1 mark]

(ii) Find an upper bound for the minimum travelling time by using the tour $MNBELM$.
[1 mark]

(iii) Write down the actual route corresponding to the tour $MNBELM$.
[2 marks]

(iv) Use the nearest-neighbour algorithm, starting from M, to find another upper bound for the minimum travelling time of Rob's tour.
[4 marks]

QUESTION PART REFERENCE	**Answer space for question 6**
(a)(i)	The best upper bound is ________________
(ii)	The best lower bound is ________________

7 A factory makes batches of three different types of battery: basic, long-life and super.

Each basic batch needs 4 minutes on machine A, 7 minutes on machine B and 14 minutes on machine C.

Each long-life batch needs 10 minutes on machine A, 14 minutes on machine B and 21 minutes on machine C.

Each super batch needs 10 minutes on machine A, 14 minutes on machine B and 28 minutes on machine C.

Machine A is available for 4 hours a day, machine B for 3.5 hours a day and machine C for 7 hours a day.

Each day the factory must make:

more basic batches than the total number of long-life and super batches;

at least as many long-life batches as super batches.

At least 15% of the production must be long-life batches.

Each day, the factory makes x basic, y long-life and z super batches.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, writing your answers with simplified integer coefficients.

[8 marks]

QUESTION PART REFERENCE

Answer space for question 7

8 In this question you may use the fact that any simple graph must have an even number of vertices of odd degree.

(a) A simple graph has five vertices and their degrees are

$$x,\ x+1\ ,\ x+1\ ,\ x+2\ \text{ and }\ x+3$$

(i) Show that x must be odd.

[2 marks]

(ii) Find the value of x and draw a graph with vertices having the given degrees.

[3 marks]

(b) A simple graph has 10 vertices.

(i) State the minimum possible degree and maximum possible degree of a vertex.

[2 marks]

(ii) Show that the degrees of the vertices cannot all be different.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 8

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
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TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2015

Mathematics MD01

Unit Decision 1

Tuesday 16 June 2015 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
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Information
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- The maximum mark for this paper is 75.

Advice
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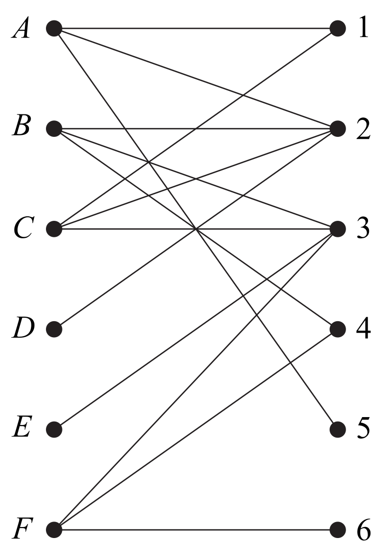
Answer **all** questions.

Answer each question in the space provided for that question.

1 A quiz team must answer questions from six different topics, numbered 1 to 6. The team has six players, A, B, C, D, E and F. Each player can only answer questions on one of the topics. The players list their preferred topics. The bipartite graph shows their choices.



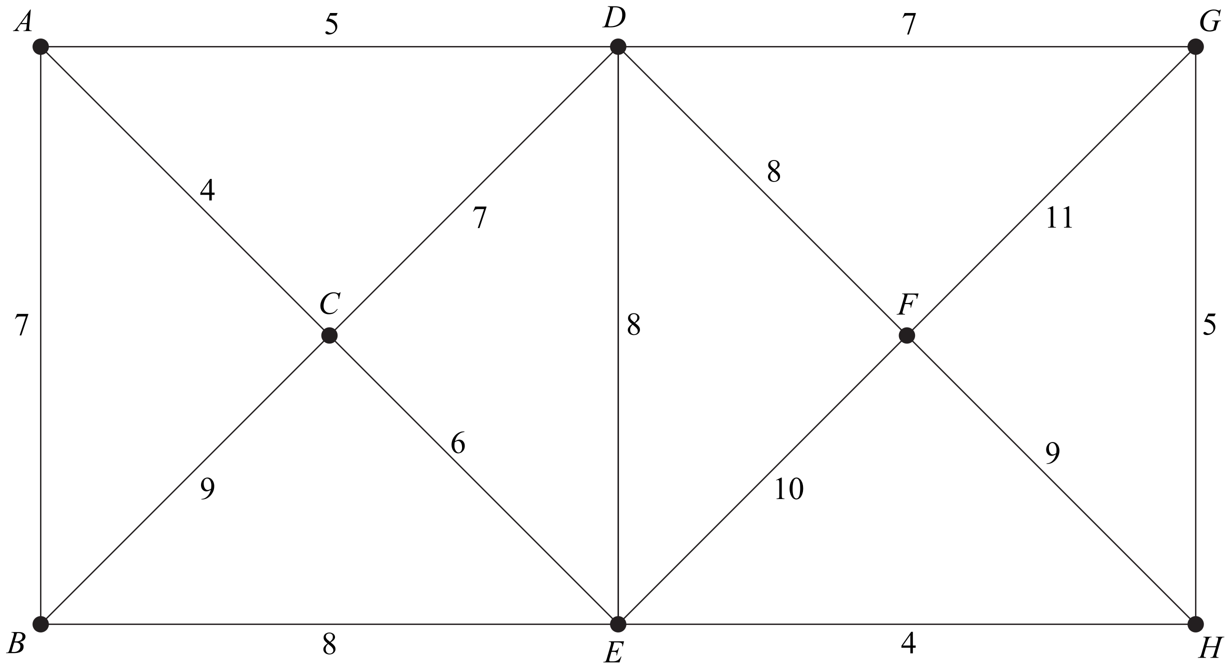
Initially, A is allocated topic 2, B is allocated topic 3, C is allocated topic 1 and F is allocated topic 4. By using an alternating path algorithm from this initial matching, find a complete matching.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

2 The network below shows 8 towns, A, B, ..., H. The number on each edge shows the length of the road, in miles, between towns. During the winter, the council treats some of the roads with salt so that each town can be safely reached on treated roads from any other town. It costs £30 to treat a mile of road.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network.

[4 marks]

(ii) Draw your minimum spanning tree.

[2 marks]

(iii) Calculate the minimum cost to the council of making it possible for each town to be safely reached on treated roads from any other town.

[1 mark]

(b) On one occasion, the road from C to E is impassable because of flooding. Find the minimum cost of treating sufficient roads for safe travel in this case.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 2

3 Four students, A, B, C and D, are using different algorithms to sort 16 numbers into ascending order.

(a) Student A uses the quicksort algorithm.

State the number of comparisons on the first pass.

[1 mark]

(b) Student B uses the Shell sort algorithm.

State the number of comparisons on the first pass.

[1 mark]

(c) Student C uses the shuttle sort algorithm.

State the minimum number of comparisons on the **final** pass.

[1 mark]

(d) Student D uses the bubble sort algorithm.

Find the maximum **total** number of comparisons.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 The network opposite shows roads connecting 10 villages, A, B, ..., J. The time taken to drive along a road is not proportional to the length of the road. The number on each edge shows the average time, in minutes, to drive along each road.

(a) A commuter wishes to drive from village A to the railway station at J.

(i) Use Dijkstra's algorithm, on the diagram opposite, to find the shortest driving time from A to J.

[5 marks]

(ii) State the corresponding route.

[1 mark]

(b) A taxi driver is in village D at $10.30\,\mathrm{am}$ when she receives a radio call asking her to pick up a passenger at village A and take him to the station at J. Assuming that it takes her 3 minutes to load the passenger and his luggage, at what time should she expect to arrive at the station?

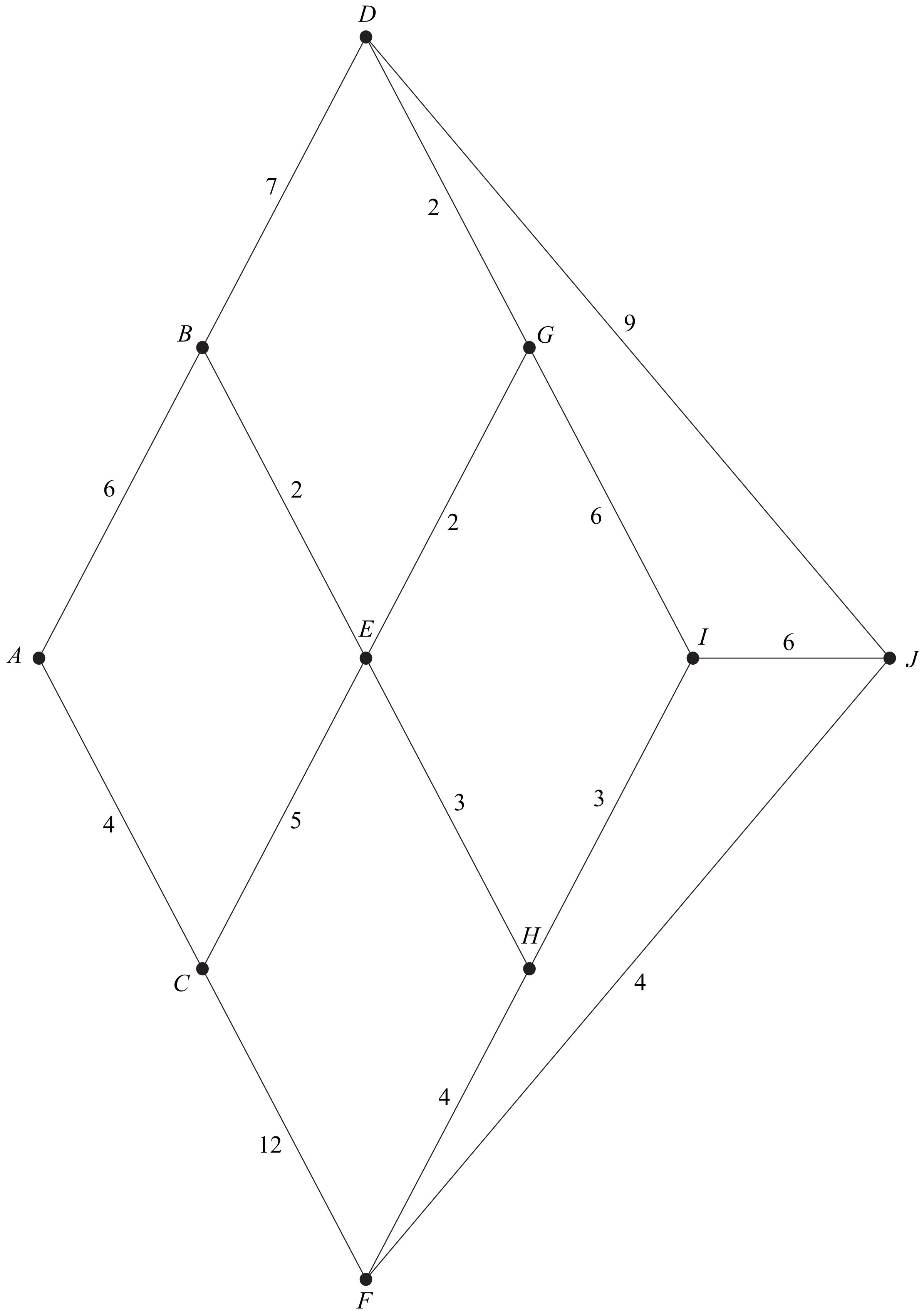
[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

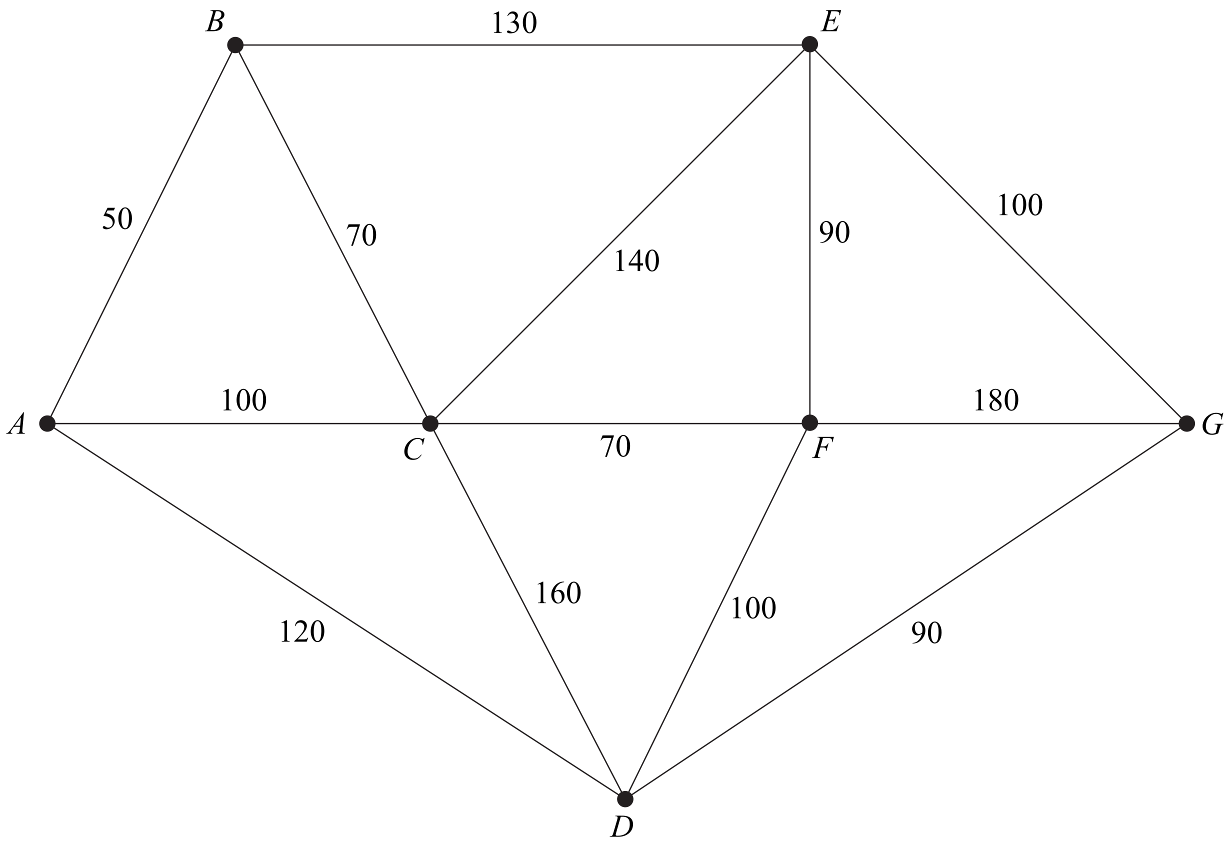
QUESTION PART REFERENCE

Answer space for question 4



Turn over ▶

5 The network shows the paths mown through a wildflower meadow so that visitors can use these paths to enjoy the flowers. The lengths of the paths are shown in metres.



The total length of all the paths is $1400\,\text{m}$.

The mower is kept in a shed at A. The groundskeeper must mow all the paths and return the mower to its shed.

(a) Find the length of an optimal Chinese postman route starting and finishing at A.

[5 marks]

(b) State the number of times that the mower, following the optimal route, will pass through:

(i) C;

[1 mark]

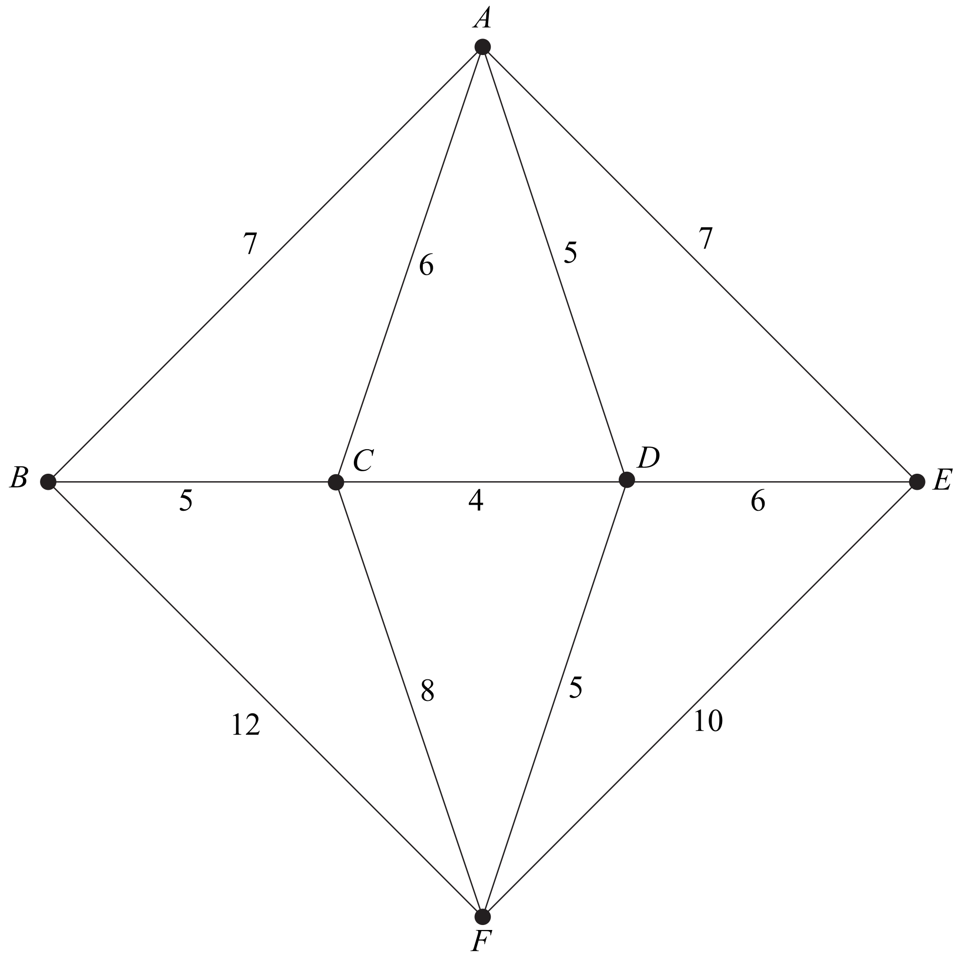
(ii) D.

[1 mark]

QUESTION PART REFERENCE	**Answer space for question 5**

Do not write outside the box

6 The network shows the roads linking a warehouse at A and five shops, B, C, D, E and F. The numbers on the edges show the lengths, in miles, of the roads.
A delivery van leaves the warehouse, delivers to each of the shops and returns to the warehouse.



(a) Complete the table, on the page opposite, showing the shortest distances between the vertices.

[2 marks]

(b) (i) Find the total distance travelled if the van follows the cycle $AEFBCDA$.

[1 mark]

(ii) Explain why your answer to part **(b)(i)** provides an upper bound for the minimum journey length.

[1 mark]

(c) Use the nearest neighbour algorithm starting from D to find a second upper bound.

[3 marks]

(d) By deleting A, find a lower bound for the minimum journey length.

[4 marks]

(e) Given that the minimum journey length is T, write down the best inequality for T that can be obtained from your answers to parts **(b)**, **(c)** and **(d)**.

[1 mark]

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

(a)

	A	B	C	D	E	F
A	–	7				
B	7	–	5			
C		5	–	4		
D			4	–	6	
E				6	–	10
F					10	–

7 (a) A simple connected graph has 4 edges and m vertices. State the possible values of m.
[2 marks]

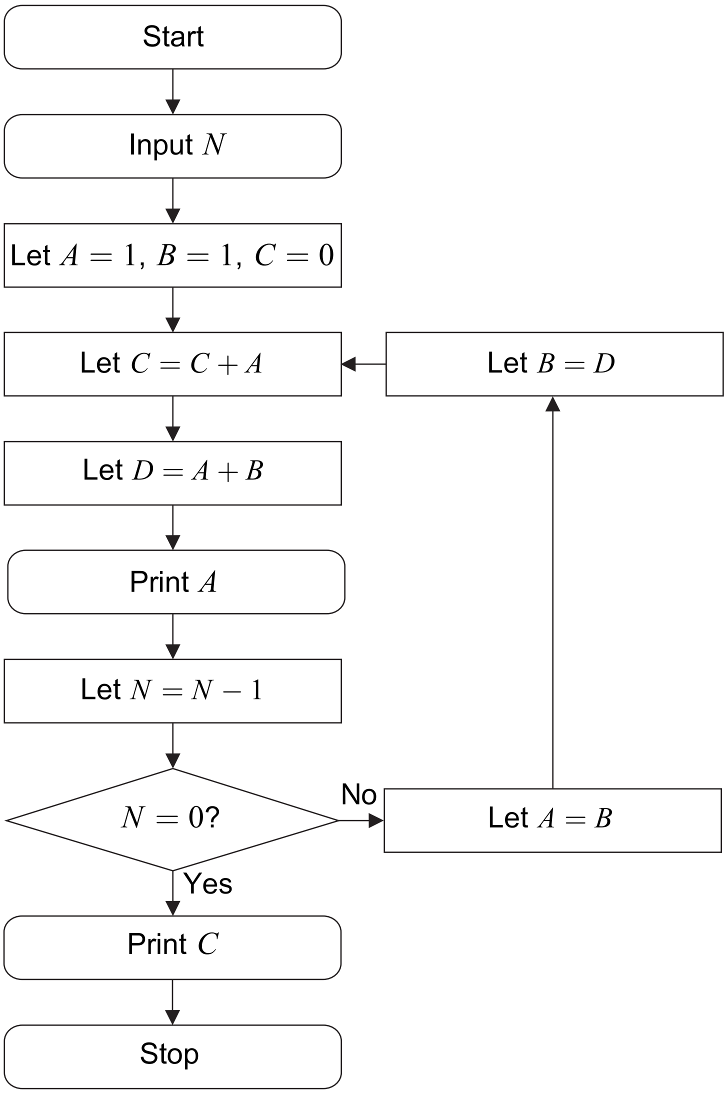
(b) A simple connected graph has n edges and 4 vertices. State the possible values of n.
[2 marks]

(c) A simple connected graph, G, has 5 vertices and is Eulerian but not Hamiltonian. Draw a possible graph G.
[2 marks]

QUESTION PART REFERENCE

Answer space for question 7

8 A student is tracing the following algorithm.



(a) Trace the algorithm illustrated in the flowchart for the case where the input value of N is 5.

[5 marks]

(b) Explain the role of N in the algorithm.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 8

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9 A company producing chicken food makes three products, Basic, Premium and Supreme, from wheat, maize and barley.

A tonne $(1000\,\text{kg})$ of Basic uses $400\,\text{kg}$ of wheat, $200\,\text{kg}$ of maize and $400\,\text{kg}$ of barley.
A tonne of Premium uses $400\,\text{kg}$ of wheat, $500\,\text{kg}$ of maize and $100\,\text{kg}$ of barley.
A tonne of Supreme uses $600\,\text{kg}$ of wheat, $200\,\text{kg}$ of maize and $200\,\text{kg}$ of barley.

The company has 130 tonnes of wheat, 70 tonnes of maize and 72 tonnes of barley available.

The company must make at least 75 tonnes of Supreme.

The company makes £50 profit per tonne of Basic, £100 per tonne of Premium and £150 per tonne of Supreme.

They plan to make x tonnes of Basic, y tonnes of Premium and z tonnes of Supreme.

(a) Write down four inequalities representing the constraints $(\text{in addition to } x,\ y \geqslant 0)$.

[4 marks]

(b) The company want exactly half the production to be Supreme.

Show that the constraints in part **(a)** become

$$\begin{aligned} x + y &\leqslant 130 \\ 4x + 7y &\leqslant 700 \\ 2x + y &\leqslant 240 \\ x + y &\geqslant 75 \\ x &\geqslant 0 \\ y &\geqslant 0 \end{aligned}$$

[2 marks]

(c) On the grid opposite, illustrate all the constraints and label the feasible region.

[5 marks]

(d) Write an expression for P, the profit for the whole production, in terms of x and y only.

[2 marks]

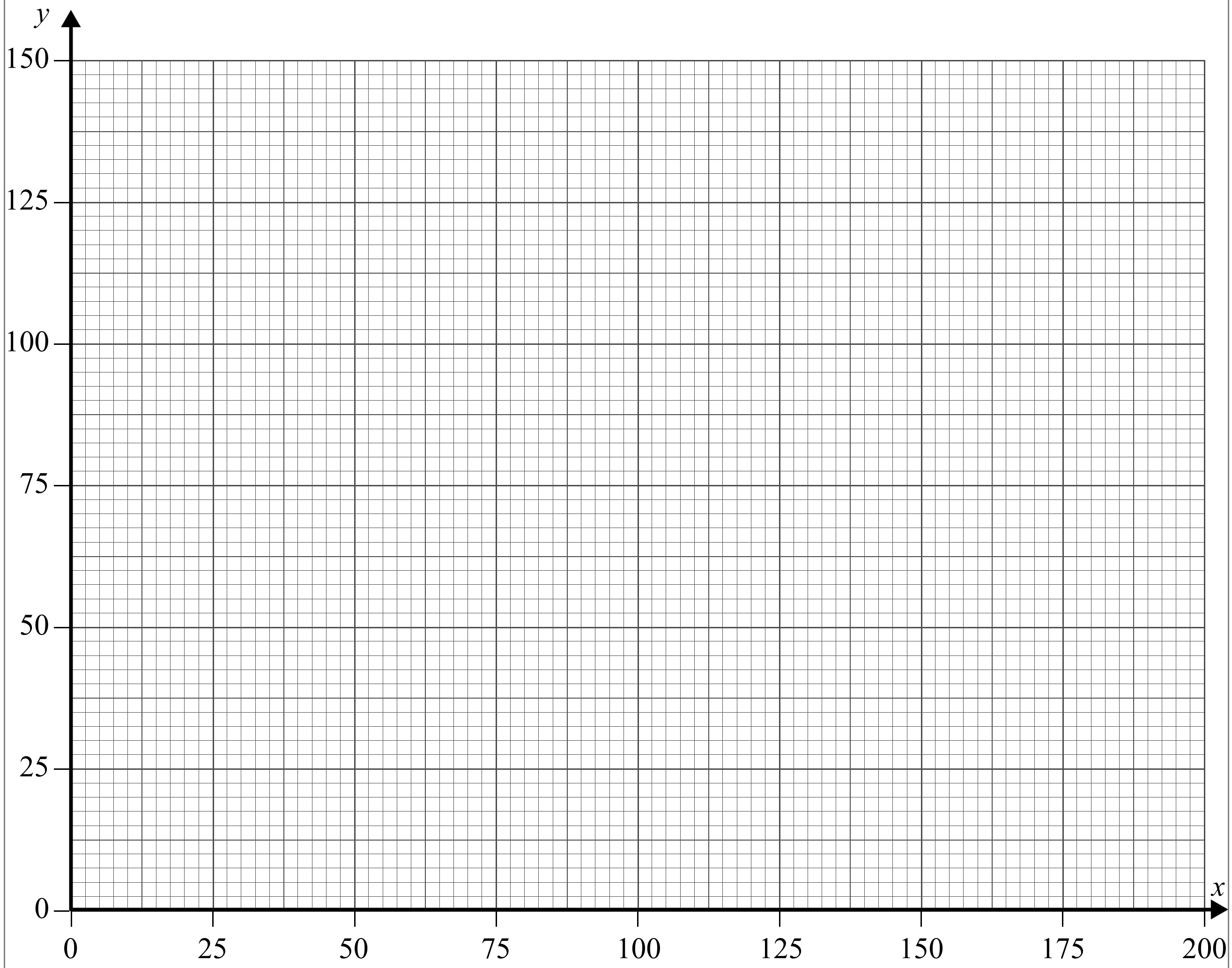
(e) (i) By drawing an objective line on your graph, or otherwise, find the values of x and y which give the maximum profit.

[2 marks]

(ii) State the maximum profit and the amount of each product that must be made.

[2 marks]

Answer space for question 9



QUESTION PART REFERENCE

Turn over ▶